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Collatz conjecture

Shubhankar Paul

Abstract— **The Collatz conjecture is an elusive problem in mathematics regarding the oneness of natural numbers when run through a specific function based on being odd or even, specifically stating that regardless of the initial number the series will eventually reach the number 1.We will prove everything for odd integer. Because even integer upon division by 2 (i.e. Collatz operation) we will eventually find a odd number, We can say for that number we are starting from there (without violating Collatz operation rule).**

Index Terms— **Collatz conjecture, odd integer.**

I. Introduction

Take any positive integer: if the number is even, divide it by two; if the number is odd, triple it and add one (for example, if this operation is performed on 26, the result is 13; if it is performed on 5, the result is 16). Perform this operation repeatedly, beginning with any positive integer, and taking the result at each step as the input at the next.

For example, starting at 11, the sequence goes 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

The Collatz conjecture is: this process will eventually reach the number 1, regardless of which positive integer is chosen initially.

The problem is to prove the conjecture, or find a counter-example.

II. Solution

Case 1 : *n is bounded above.*

n is odd.

n will not increases infinitely if we get more even number after doing 3n+1.

Let's take worst possible scenario → When every time we do 3n+1 it gets divided by 2 then it becomes odd.

So, according to our assumption (3n+1)/2 is odd.

Now we are doing $(3n+1)/2 = n + (n+1)/2$.

n is an integer. So, (n+1)/2 must be an integer => (n+1) is divisible by 2

Again doing so we get, $n + (n+1)/2 + 3(n+1)/2^2$

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Shubhankar Paul, Passed BE in Electrical Engineering from Jadavpur University in 2007. Worked at IBM as Manual Tester with designation Application Consultant for 3 years 4 months. Worked at IIT Bombay for 3 months as JRF.

n and (n+1)/2 are integer. So, $3(n+1)/2^2$ must be an integer => (n+1) is divisible by 2^2

Again doing so we get, $n + (n+1)/2 + 3(n+1)/2^2 + 3^2(n+1)/2^3$

n, (n+1)/2, $3(n+1)/2^2$ are integers. So, $3^2(n+1)/2^3$ must be an integer => (n+1) is divisible by 2^3

Again doing so we get, $n + (n+1)/2 + 3(n+1)/2^2 + 3^2(n+1)/2^3 + 3^3(n+1)/2^4$

n, (n+1)/2, $3(n+1)2^2$, $3^2(n+1)/2^3$ are integers. So, $3^3(n+1)/2^4$ must be an integer => (n+1) is divisible by 2^4 .

In this way continuing we can say that n+1 must be of the form 2^a to go on increasing infinitely.

Now, n = 2^a-1

Doing Collatz operation (3n +1)/2 = {3*2^a – 3 +1}/2 = 3*2^(a-1) – 1 which is odd.

Again doing (3n+1)/2 gives 3^2*2^(a-2) – 1 which is again odd.

Again doing (3n+1)/2 gives 3^3*2^(a-3) -1 which is again odd.

So, continuing in this way once we will find 3^(a-1)*2 – 1 which is again odd.

Next doing (3n+1)/2 gives 3^a – 1 which is even.

(Contradiction)

So, it is divisible by 2.

Here is contradiction that it goes on increasing infinitely.

So, n is bounded with the operation (3n+1) if n is odd and n/2 is n is even.

Case 1 proved.

Now, n can move into a cycle. Let's say n goes to m then after r operation n again come to m

i.e. m is becoming m again after doing operation (3n+1) if n is odd and n/2 is n is even.

e.g. if we do 5n+1 when n is odd and n/2 when n is even on 27 then we get as below :

27 = 27*5+1 = 136
136/2 = 68
68/2 = 34
34/2 = 17
17*5+1 = 86
86/2 = 43
43*5+1 = 216
216/2 = 108
108/2 = 54
54/2 = 27

So, we see that 27 is again 27 after doing 5n+1 when n is odd and n/2 when n is even.

Case 2 : *No cycle exist if we do 3n+1 if n is odd and n/2 if n is even on n.*

Let's say n runs into a cycle from m to m.

m is odd.

There must be an integer of the form (2^r)*m in between.

Let's say just the previous number before (2^r)*m is b

b is odd.

Therefore, 3b+1 = (2^r)*m

⇨ b = {(2^r)*m -1}/3

Now, $2 \equiv -1 \pmod 3$

- ⇨ 2^r ≡ (-1)^r (mod 3)
- ⇨ {(2^r)*m -1 ≡ m*(-1)^r -1 (mod 3) = (-1)^r*m – 1 (mod 3)

And (-1)^r*m – 1 must be ≡ 0 (mod 3) as b is integer.
- ⇨ m ≡ (-1)^r (mod 3)
- ⇨ m = 3s +(-1)^r

Now, s must be of the form 2q i.e. even because m is odd.
- ⇨ m = 6q + (-1)^r

Now, b = {(2^r)*m -1}/3
- ⇨ b = {(2^r)*(6q+(-1)^r) -1}/3
- ⇨ b = 2^(r+1)*q + {2^r(-1)^r – 1}/3

Now, 2^r(-1)^r – 1= 3p
- ⇨ 2^r = (3p+1)/(-1)^r
- ⇨ r is even and p is odd
- ⇨ 4^s = (6i+4) where s=2r and p = 2i+1
- ⇨ 2*4^(s-1) = 3i+2
- ⇨ i = 2{4^(s-1) – 1}/3
- ⇨ i is even = 2j (say)

Now, 2*4^(s-1) = 6j + 2
- ⇨ 4^(s-1) = 3j + 2

j must be even = 2k (say)
Now, 4^(s-1) = 6k+2
- ⇨ 2*4^(s-1) = 3k+1

k must be odd = 2d+1 (say)
Now, 2*4^(s-1) = 6d+4
- ⇨ 4^(s-1) = 3d+2

If we go on doing this way once we will find an equation like 4 = 3w+2 which is impossible.
Here is the contradiction.
So, a number cannot move into a cycle.
Case 2 proved.

Observation :
1) From Case 1 we have seen every number is bounded if we do 3n+1 if n is odd and n/2 if n is even.
2) From Case 2 we have seen that a number cannot move into a cycle.

III. Result :

Collatz Conjecture is true.
End.

IV. Conclusion

So, every number will eventually come to least natural number 1 if we do Collatz operation on it.

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Shubhankar Paul, Passed BE in Electrical Engineering from Jadavpur University in 2007. Worked at IBM as Manual Tester with designation Application Consultant for 3 years 4 months. Worked at IIT Bombay for 3 months as JRF. Published 2 papers at International Journal