Twin Prime Conjecture Proof

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Abstract— **A number is called prime when the number is divisible by 1 and that number itself, no other factor. If two consecutive odd numbers are prime then they are called twin prime.**

Index Terms— **prime, Polymath project**

I. Introduction

Twin primes are pairs of primes of the forms $(p, p+2)$. The term "twin prime" was coined by Paul Stäckel (1862-1919; Tietze 1965, p. 19). The first few twin primes are $n \pm 1$ for $n = 4$, 6, 12, 18, 30, 42, 60, 72, 102, 108, 138, 150, 180, 192, 198, 228, 240, 270, 282, ... Explicitly, these are (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), ...

It is conjectured that there are an infinite number of twin primes (this is one form of the twin prime conjecture), but proving this remains one of the most elusive open problems in number theory.

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states: There are infinitely many primes p such that p + 2 is also prime. In 1849 de Polignac made the more general conjecture that for every natural number k, there are infinitely many prime pairs p and p′ such that p′ − p = 2k. The case k = 1 is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture, postulates a distribution law for twin primes akin to the prime number theorem.

On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is at most 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath project collaborative effort to optimize Zhang's bound; as of July 20, 2013, the Polymath project participants claim to have reduced the bound to 5,414.

Largest known twin prime pair :

On January 15, 2007 two distributed computing projects, Twin Prime Search and PrimeGrid found the largest known twin primes, 2003663613 · 2^195000 ± 1. The numbers have 58711 decimal digits. Their discoverer was Eric Vautier of France.

On August 6, 2009 those same two projects announced that a new record twin prime had been found. It is 65516468355 · 2^333333 ± 1. The numbers have 100355 decimal digits.

On December 25, 2011 PrimeGrid announced that yet another record twin prime had been found. It is 3756801695685 · 2^666669 ± 1. The numbers have 200700 decimal digits.

An empirical analysis of all prime pairs up to 4.35 · 10^15 shows that if the number of such pairs less than x is f(x)·x/(log x)^2 then f(x) is about 1.7 for small x and decreases towards about 1.3 as x tends to infinity.

There are 808,675,888,577,436 twin prime pairs below 10^18

Solution :

Twin primes are of the form 6n±1 except (3,5). As of now we are not thinking about (3,5) as we are interested in infinite number of twin prime exists or not. Now I call a number twin prime generator if it generated a twin prime. For example 12 is a twin prime generator because 11 and 13 constitute twin prime. So, 6n is twin prime generator.

Now, if we divide 6n by 5 we find that the generator's last digit cannot be 4 or 6 because otherwise 4's next number and 6's previous number is divisible by 5 and cannot generate twin prime. So, last digit of twin primes can be wither 0, 2, or 8.

Now, working with 0. 10, 20 are not twin prime generator because they are not divisible by 3. So, either previous one or next number will be divided by 3 not generating twin primes. for example here, 9 and 21. Twin prime generator must be divisible by 3 so it takes form 6n. So 30 is a twin prime generator. so, twin prime generator is of the form 30n + 30 except (3,5); (5,7); (11,13); (17,19). As of now we are not thinking about those as we are interested in infinite number of twin prime exists or not.

Now, let's divide 30n+30 by 7. If it doesn't give +1 or -1 as remainder then it is a twin prime generator w.r.t. 7. If it gives +1 or -1 as remainder then the previous number or the next number will be divisible by 7 and not a twin prime generator.

Now, if we divide 30(n+1) by 7 then n can be 2, 9, 16, 23,.... which is a series of common difference 7 which gives -1 as a remainder & n can be 3, 10, 17, 24.......which is a series of common difference 7 again which gives 1 as a remainder. Let's call the two sets together {7}

Similarly. if we divide it by 11 then series is 6,17, 28,...which is series of common difference 11 which gives 1 as remainder & 3, 14, 25,......which is series of common difference 11 which gives -1 as remainder. Let's call the two sets together {11}

Similarly for 13 the series is 2, 15, 28,.....for -1 & series is 9, 22, 35......for 1. Let's call the two sets together {13}.

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Similarly. we can find for other prime numbers which on division of 30(n+1) gives ±1 as remainder.
Now as we see the prime number is increasing (property of natural number) so the common difference will also increase.
Now we need to find numbers which are not part of these series taken simultaneously.
As the prime number series diverges as it goes on increasing then there must be some integers which are not part of these series. So that we can find n and substitute to get a twin prime generator. Once twin prime generator is found then twin primes can be found.
If the numbers which gives remainder ±1 is called set {n} = {7}υ{11}υ{13}υ....υ then {n} must be subset of {Z} the set of integers. {Z}-{n} gives the n's for which 30 + 30n is a generator of twin prime. Exclude the numbers which gives ±1 as remainder with quotient 1 because they are prime.
Obviously {z}-{n} is non-empty because 1 is an element of the set as 59 and 61 twin prime itself. And {Z}-{n} is infinite as the series of {n} continues to go on so we will find corresponding {Z}-{n}.
This similar case also goes with the numbers 12 + 30n and 18 + 30n.

II. RESULT

Twin primes are infinite.

III. CONCLUSION

The Twin Prime Conjecture is true.

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