Exponential attractor for the Higher-order Kirchhoff-type equation with nonlinear strongly damped term

Guoguang Lin, Yuting Sun, Yunlong Gao

***Abstract*— We investigate the existence of exponential attractor for the Higher-order Kirchhoff-type equation with nonlinear strongly damped term:**

$$u_{tt}+\sigma\left(\|\nabla^m u\|^2\right)(-\Delta)^m u_t+\phi\left(\|\nabla^m u\|^2\right)(-\Delta)^m u=f(x)$$

.For strong nonlinear damping $\sigma(s)$ and $\phi(s)$,we assumptions $(H_1)-(H_3)$.Under of the proper assume, we first prove the squeezing property of the nonlinear semigroup associated with this equation, then the existence of exponential attractor is proved.

***Index Terms*— Higher-order, squeezing property, Exponential attractor.**

I. INTRODUCTION

In this paper, we study the existence of exponential attractor for the Higher-order Kirchhoff-type equation with nonlinear strongly damped term:

$$u_{tt}+\sigma(\|\nabla^m u\|^2)(-\Delta)^m u_t+\phi(\|\nabla^m u\|^2)(-\Delta)^m u=f(x),(x,t)\in\Omega\times[0,+\infty), \tag{1.1}$$

$$u(x,t)=0,\ \frac{\partial^i u}{\partial \nu^i}=0,\ i=1,2,\dots,m-1,\ x\in\partial\Omega, t\in(0,+\infty), \tag{1.2}$$

$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x),\ x\in\Omega. \tag{1.3}$$

where Ω is a bounded domain of R^n , with a smooth dirichlet boundary $\partial\Omega$ and initial value, and $m>1$ is an integer constant. Moreover, ν is the unit outward normal on $\partial\Omega$. $\sigma(s)$ and $\phi(s)$ are scalar functions specified later, f is a given function.

The exponential attractor, in contrast to a global attractor, enjoys a uniform exponential rate of convergence of its solutions once the solution is invariant absorbing set, and it is an important feature of the long time behaviors of nonlinear partial differential equations. because of this, exponential attractors possess a deeper and more practical property, and they remain more robust under perturbations and numerical approximations than global attractors[1-2]. Hence, since the concept of Foias [3] was proposed in 1994, many authors began to study the existence of exponential and numerical approximations than global attractors; see[4-11].

Recently, Wenting Wang and Qiaozhen Ma[4] research the existence of the exponential attractors for the suspension bridge equation.

$$\begin{cases} u_{tt}-\Delta^2 u+\delta u_t+ku^++g(u)=h, & (x,t)\in\Omega\times R^+ \\ u=\Delta u=0, & x\in\Gamma \\ u(x,0)=u_1(x),u_t(x,0)=u_2(x). & x\in\Omega. \end{cases} \tag{1.4}$$

The suspension bridge equation is an important model in engineering mathematics. We have a lot of results for attractors in class of autonomous and non-autonomous conditions. However, there are nobody have discussed the problem of exponential attractors for this problem. This paper, based on the methods proposed by Aden et al, they proved the existence of the exponential attractors for the suspension bridge equation.

In 2005, Zhengde Dai and Dacai Ma[5] study the exponential attractors for the following nonlinear wave equations:

$$\begin{cases} u_{tt}+\alpha u_t-\Delta u+g(u)=f(x), & (x,t)\in\Omega\times R^+ \\ u=0, & (x,t)\in\partial\Omega\times R^+ \\ u(x,0)=u_0(x),u_t(x,0)=u_1(x). \end{cases} \tag{1.5}$$

Then in 2012, Jundong Jin, Jianhua Ding and Wanxiong Wang$^{[6]}$ used the same method and the existence of exponential attractors for the generalized convection-diffusion expansion equation is obtained by using the decomposing technique of operator semigroup in $L^2(\Omega)$.

$$\begin{cases} u_t-\Delta u+f(u)=0 \\ u(0)=u_0,u(x,t)\big|_{\partial\Omega}=0. \end{cases} \tag{1.6}$$

In 2015, Zhijian Yang and Zhiming Liu studied the existence of exponential attractor for the Kirchhoff equations with strong nonlinear damping and supercritical nonlinearity

Guoguang Lin , Yuting Sun, Yunlong Gao, Mathematics of Yunnan University, Kunming, Yunnan 650091, People's Republic of China

$$u_{tt}-\sigma(\|\nabla u\|^2)(\Delta)u_t-\phi(\|\nabla u\|^2)(\Delta)u+f(u)=h(u), x\in\Omega, t>0.$$
$$u|_{\partial\Omega}=0, u(x,0)=u_0(x), u_t(x,0)=u_1(x), x\in\Omega \quad (1.7)$$

Where Ω is a bounded domain of R^n, with a smooth boundary $\partial\Omega$, the nonlinear functions $\sigma(s)$, $\phi(s)$ and $f(u)$ and the external force term h will be specified later. The main result is that the nonlinearity $f(u)$ is of supercritical growth. In this case, we establish an exponential attractor in natural energy space by using a new method based on the weak quasi-stability estimates (rather than the strong one as usual.)

At present, most Higher-order Kirchhoff-type equations investigate the blow-up of the solution. On the basis of Yang, we investigate the exponential attractor of the higher-order Kirchhoff-type equation (1.1) with strong nonlinear damping. Such problems have been studied by many authors, but $\sigma(\|\nabla^m u\|^2)$ is a definite constant and even $\sigma(\|\nabla^m u\|^2)=0$. Generally, the equation exist a nonlinear $f(u)$. But in the paper, $\sigma(\|\nabla^m u\|^2)$ is a scalar function and $f(u)=0$. Under of the the proper assume, in section 2, we introduce some basic concepts. In section 3, we prove the squeezing property of the nonlinear semigroup associated with this equation, then the existence of existence of exponential attractor is proved.

II. Preliminaries

For brevity, we denote the simple symbol, $\|\cdot\|$ represents inner product,

$H^m=H^m(\Omega)$, $H_0^m=H_0^m(\Omega)$, $H^{2m}=H^{2m}(\Omega)$, $H=L^2$, $\|\cdot\|=\|\cdot\|_{L^2}$,

$\|\cdot\|_\infty=\|\cdot\|_{L^\infty}$, $f=f(x)$, $c_i(i=1,2)$ are constants, $m_i(i=0,1)$ are also constants. λ^m is the first eigenvalue of the operator ∇^m. The notation $(\cdot,\cdot)$ for the H-inner product will also be used for the notation of duality pairing between dual spaces.

In this section, we present some assumptions needed in the proof of our results, state a global existence result, and prove our main result. For this reason, we assume that

(H_1) $\phi(s)\in C^1$ $\phi(s)>0, \forall s>0;$ (2.1)

(H_2) $\sigma(s)\in C^1$ $\sigma(s)>0, \forall s>0;$ (2.2)

(H_3) $1<m_0<\phi(s)-\varepsilon\sigma(s)<m_1, \forall s>0.$ (2.3)

We will use the following notations. Let V_1, V_2 are two Hilbert spaces, we have $V_2\subset V_1$ with dense and continuous injection, and $V_2\subset V_1$ is compact. Let $S(t)$ is a map from $V_1(V_2)$ into $V_1(V_2)$ [11].

Definition 2.1 The semigroup $S(t)$ possesses a (V_2,V_1)-compact attractor $\mathcal{A}$, If it exists a compact set $\mathcal{A}\subset V_1$, $\mathcal{A}$ attracts all bounded subsets of V_2, and $S(t)\mathcal{A}=\mathcal{A}, \forall t\geq 0$.

Definition 2.2 A compact set $\mathcal{M}$ is called a $(V_2,V_1)-$exponential attractor for the system $(S(t),B)$, if $\mathcal{A}\subseteq\mathcal{M}\subseteq B$ and

1) $S(t)\mathcal{M}\subseteq\mathcal{M}$, $\forall t\geq 0$,

2) $\mathcal{M}$ has finite fractal dimension, $d_F(\mathcal{M})<+\infty$,

3) there exist universal constants c_1, c_2 such that

$$dist(S(t)B,\mathcal{M})\leq c_1 e^{-c_2 t}, \quad t>0, \quad (2.4)$$

where $dist_{V_1}(A,B)=\sup_{x\in A}\inf_{y\in B}|x-y|_{V_1}$, $B\subset V_1$ is the positive invariant set of $S(t)$.

Definition 2.3 $S(t)$ is said to satisfy the discrete squeezing property on B if there exists an orthogonal projection P_N of rank N such that for every u and v in B,

$$|S(t_*)u-S(t_*)v|_{V_1}\leq\delta|u-v|_{V_1}, \quad \delta\in(0,1/8), \quad (2.5)$$

or

$$|Q_N(S(t_*)u-S(t_*)v)|_{V_1}\leq|P_N(S(t_*)u-S(t_*)v)|_{V_1} \quad (2.6)$$

where $Q_N=I-P_N$.

Theorem 2.1 [11] Assume that

1) $S(t)$ possesses a (V_2,V_1)-compact attractor $\mathcal{A}$;

2) it exists a positive invariant compact set $B\subset V_1$ of $S(t)$;

3) $S(t)$ is a Lipschitz continuous map with Lipschitz constant l on B, and satisfies the discrete squeezing property on B (with rank N_0).

Then $S(t)$ has a $(V_2,V_1)-$exponential attractor $\mathcal{M}\supseteq\mathcal{A}$ on B, and

$$\mathcal{M}=\bigcup_{0\leq t\leq t_*}S(t)\mathcal{M}_* \quad (2.7)$$

where

$$\mathcal{M}_*=\mathcal{A}\cup\left(\bigcup_{j=1}^{\infty}\bigcup_{k=1}^{\infty}S(t_*)^j(E^{(k)})\right). \quad (2.8)$$

Moreover, the fractal dimension of $\mathcal{M}$ satisfies

$$d_F(\mathcal{M})\leq N_0\max\left\{1,\frac{\log(16l+1)}{2\log 2}\right\}, \quad (2.9)$$

$$dist_{V_1}(S(t)B,\mathcal{M})\leq c_1 e^{-c_2 t}, \quad (2.10)$$

where $\theta, N_0, E^{(k)}$ are the same as [12] definition, l is Lipschitz constant for $S(t)$ in B.

Proposition 2.1[11] There is $t_0(B_0)$ such that $B=\overline{\bigcup_{0\le t\le t_0(B_0)} S(t)B_0}$ is the positive invariant set of $S(t)$ in V_1, and B attracts all bounded subsets of V_2, where B_0 is a closed bounded absorbing set for $S(t)$ in V_2.

Proposition 2.2 Let B_0, B_1 respectively are closed bounded absorbing set of problem (1.1)-(1.3) in V_2, V_1, then $S(t)$ possesses a (V_2,V_1)-compact attractor A.

Remark 1 The proof of the Proposition 2.1 and Proposition 2.2 refer to [14].

III. Exponential attractors

Let $V_1 = H_0^m \times H$ endowed with the inner product and norm as: for any $U_i = (u_i, v_i) \in V_1, i = 1,2$.

$$(U_1,U_2)_{V_1} = (\Delta^m u_1, \Delta^m u_2) + (v_1, v_2),$$

(3.1)

$$\| U\|_{V_1}^2 = (U,U)_{V_1} = \| \Delta^m u\|^2 + \| v\|^2, \quad U \in V_1$$

(3.2)

Let $v = u_t + \varepsilon u$,

and

$$\frac{(m_0-1)\lambda^m - 1}{2\lambda^m} < \varepsilon < mim\left\{ \frac{\sigma\left(\|\nabla^m u\|^2\right)\lambda^m}{2}, \sqrt[4]{\sigma\left(\|\nabla^m u\|^2\right)+1-m_0}\cdot\lambda^{\frac{m}{4}} \right\},$$

,then problem (1.1)-(1.3) can be writen as

$$\mathrm{U}_t + \mathrm{H(U)} = \mathrm{F(x)} \tag{3.3}$$

where

$$H(U) = \begin{pmatrix} \varepsilon u - v \\ -\varepsilon v + \varepsilon^2 u + \sigma\left(\|\nabla^m u\|^2\right)(-\Delta)^m v + \left(\phi\left(\|\nabla^m u\|^2\right) - \varepsilon\sigma\left(\|\nabla^m u\|^2\right)\right)(-\Delta)^m u \end{pmatrix} \tag{3.4}$$

$$F(X) = \begin{pmatrix} 0 \\ f(x) \end{pmatrix} \tag{3.5}$$

Theorem 3.1[13] Let $(H_1)-(H_3)$ be in force, assume that $f \in H$, $(u_0,v_0) \in V_k, k = 1,2$, then problem(1.1)-(1.3)admits a unique solution $(u,v) \in L^\infty(\square^+; V_k)$. This solution possesses the following properties:

$$\|(u,v)\|_{V_1}^2 = \| \nabla^m u\|^2 + \| v\|^2 \le R_1, \quad \|(u,v)\|_{V_2}^2 = \|\square^m u\|^2 + \| \nabla^m v\|^2 \le R_2, \quad t \ge t_k, \tag{3.6}$$

We denote the solution in Theorem 3.1 by $S(t)(u_0,v_0) = (u(t),v(t))$. Then $S(t)$ composes a continuous semigroup in V_1.

According to Theorem 3.1, we have the ball

$$B_k = \{(u,v) \in V_k : \|(u,v)\|_{V_k} \le \sqrt{R_k}\} \tag{3.7}$$

is a absorbing set of $S(t)$ in $V_k, k = 1,2$.

From Proposition 2.1, we have

$$B = \overline{\bigcup_{0\le t\le t_0(B_2)} S(t)B_2} \tag{3.8}$$

is a positive invariant compact set of $S(t)$ in V_1, and absorbs all of the bounded subsets of V_2. According to literature[13] and Theorem 2.1, we can get the semigroup $\{S(t)\}_{t\ge 0}$ possesses $(V_2,V_1)-$compact global attractor

$$\mathrm{A} = \bigcap_{s\ge 0}\overline{\bigcup_{t\ge s} S(t)B_2}, \tag{3.9}$$

where the bar means the closure in V_1, and A is bounded in V_2.

Lemma 3.1 For any $U = (u,v) \in V_1$, we get

$$(H(U),U)_{V_1} \ge k_1 \|U\|_{V_1}^2 + k_2 \|\nabla^m v\|^2, \tag{3.10}$$

where

$$k_1 = \min\left\{ \left(\varepsilon - \frac{m_0-1}{2} - \frac{1}{2\lambda^m}\right), \left(\frac{\sigma\left(\|\nabla^m u\|^2\right)\lambda^m}{2} - \varepsilon\right) \right\} \ge 0,$$

$$k_2 = \left(\frac{\sigma\left(\|\nabla^m u\|^2\right)}{2} - \frac{m_0-1}{2} - \frac{\varepsilon}{2\lambda^m} \right) \ge 0.$$

Proof By (3.1) and (3.2), we have

$$\begin{aligned}
&(H(U),U)_{V_1} \\
&= \varepsilon\|\nabla^m u\|^2 - \varepsilon\|v\|^2 + \sigma\left(\|\nabla^m u\|^2\right)\|\nabla^m v\|^2 \\
&+ \left(\phi\left(\|\nabla^m u\|^2\right) - \varepsilon\sigma\left(\|\nabla^m u\|^2\right) - 1\right)(\nabla^m u, \nabla^m v) + \varepsilon^2(u,v) \\
&= \varepsilon\|\nabla^m u\|^2 + \frac{\sigma\left(\|\nabla^m u\|^2\right)\lambda^m}{2}\|v\|^2 + \frac{\sigma\left(\|\nabla^m u\|^2\right)}{2}\|\nabla^m v\|^2 \\
&+ \left(\phi\left(\|\nabla^m u\|^2\right) - \varepsilon\sigma\left(\|\nabla^m u\|^2\right) - 1\right)(\nabla^m u, \nabla^m v) + \varepsilon^2(u,v)
\end{aligned} \tag{3.11}$$

By the Hölder's inequality, Young's inequality and Poincarés inequality and (H_3), we receive

$$\begin{aligned}
&\left(\phi\left(\|\nabla^m u\|^2\right) - \varepsilon\sigma\left(\|\nabla^m u\|^2\right) - 1\right)(\nabla^m u, \nabla^m v) \\
&\ge (m_0 - 1)(\nabla^m u, \nabla^m v) \\
&\ge -\frac{m_0-1}{2}\|\nabla^m u\|^2 - \frac{m_0-1}{2}\|\nabla^m v\|^2,
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\varepsilon^2(u,v) &\ge -\varepsilon^2\left(\|u\|\cdot\|v\|\right) \\
&\ge -\varepsilon^2\left(\frac{1}{2\varepsilon^2}\|u\|^2 + \frac{\varepsilon^2}{2}\|v\|^2\right) \\
&= -\frac{1}{2}\|u\|^2 - \frac{\varepsilon^4}{2}\|v\|^2 \\
&\ge -\frac{1}{2\lambda^m}\|\nabla^m u\|^2 - \frac{\varepsilon^4}{2\lambda^m}\|\nabla^m v\|^2.
\end{aligned} \tag{3.13}$$

Substitution (3.12), (3.13) into (3.11), we get

$$\left(H(U),U\right)_{V_1} \geq \left(\varepsilon - \frac{m_0-1}{2} - \frac{1}{2\lambda^m}\right)\left\|\nabla^m u\right\|^2 + \left(\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)\lambda^m}{2} - \varepsilon\right)\left\|v\right\|^2 + \left(\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)}{2} - \frac{m_0-1}{2} - \frac{\varepsilon^4}{2\lambda^m}\right)\left\|\nabla^m v\right\|^2. \tag{3.14}$$

Because of

$$\frac{(m_0-1)\lambda^m - 1}{2\lambda^m} < \varepsilon < mim\left\{\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)\lambda^m}{2}, \sqrt[4]{\sigma\left(\left\|\nabla^m u\right\|^2\right) + 1 - m_0} \cdot \lambda^{\frac{m}{4}}\right\},,$$

Then

$$\varepsilon - \frac{m_0-1}{2} - \frac{1}{2\lambda^m} > 0, \quad \frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)\lambda^m}{2} - \varepsilon > 0,$$

$$\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)}{2} - \frac{m_0-1}{2} - \frac{\varepsilon^4}{2\lambda^m} > 0.$$

Hence,

$$(H(U),U)_{V_1} \geq k_1\left\|U\right\|_{V_1}^2 + k_2\left\|\nabla^m v\right\|^2. \tag{3.15}$$

where $k_1 = \min\left\{\left(\varepsilon - \frac{m_0-1}{2} - \frac{1}{2\lambda^m}\right), \left(\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)\lambda^m}{2} - \varepsilon\right)\right\} \geq 0,$

$$k_2 = \left(\frac{\sigma\left(\left\|\nabla^m u\right\|^2\right)}{2} - \frac{m_0-1}{2} - \frac{\varepsilon}{2\lambda^m}\right) \geq 0.$$

Let $S(t)U_0 = U(t) = (u(t), v(t))^T$ where $v = u_t(t) + \varepsilon u(t)$,

and $S(t)V_0 = V(t) = (\tilde{u}(t), \tilde{v}(t))^T$ where $\tilde{v}(t) = \tilde{u}_t(t) + \varepsilon\tilde{u}(t)$.

Let $W(t) = S(t)U_0 - S(t)V_0 = U(t) - V(t) = (w(t), z(t))^T$,

where $z(t) = w_t(t) + \varepsilon w(t)$.

Then $W(t)$ satisfies

$$W_t + H(U) - H(V) = 0, \tag{3.16}$$

$$W(0) = U_0 - V_0. \tag{3.17}$$

For the sake of attesting (1.1)-(1.3) has a exponential attractor, we first prove the dynamical system $S(t)$ is Lipschitz continuous on B.

Lemma 3.2 (Lipschitz property). For any $U_0, V_0 \in B$ and $T \geq 0$,

$$\| S(T)U_0 - S(T)V_0\|_{V_1} \leq e^{-2k_1 t}\| U_0 - V_0\|_{V_1}. \tag{3.18}$$

Proof Similar to Lemma 3.1, we have

$$(H(U) - H(V), W(t))_{V_1} \geq k_1\left\|W(t)\right\|_{V_1}^2 + k_2\left\|\nabla^m z\right\|^2 \tag{3.19}$$

Applying the inner product of the equation (3.16) with $W(t)$ in V_1, we discover that

$$\frac{1}{2}\frac{d}{dt}\left\|W(t)\right\|^2 + \left(H(U) - H(V), W(t)\right) = 0. \tag{3.20}$$

Collecting with (3.19), we obtain from (3.20) that

$$\frac{d}{dt}\left\|W(t)\right\|^2 + 2k_1\left\|W(t)\right\|_{V_1}^2 + 2k_2\left\|\nabla^m z\right\|^2 \le 0. \qquad (3.21)$$

Which is

$$\frac{d}{dt}\left\|W(t)\right\|^2 + 2k_1\left\|W(t)\right\|_{V_1}^2 \le 0. \qquad (3.22)$$

By using Gronwall inequality, we end up with

$$\left\|W(t)\right\|^2 \le e^{-2k_1 t}\left\|W(0)\right\|^2. \qquad (3.23)$$

So we have

$$\| S(T)U_0 - S(T)V_0\|_{V_1} \le e^{-2k_1 t}\| U_0 - V_0\|_{V_1}. \qquad (3.24)$$

Now, we define the operator $A = (-\Delta)^m : D(A) \to H$, $D(A) = \{u \mid Au \in H\}$

Obviously, A is an unbounded self-adjoint closed positive operator, and A^{-1} is compact, we find by elementary spectral theory the existence of an orthonormal basis of H consisting of eigenvectors ϖ_j of A:

$$\begin{cases} A\varpi_j = \lambda_j \varpi_j, & j = 1, 2, \cdots, \\ 0 < \lambda_1 \le \lambda_2 \le \cdots, & \lambda_j \to \infty \quad as \quad j \to \infty. \end{cases} \qquad (3.25)$$

For a given integer n we denote by P_n the orthogonal projection of H onto the space spanned by $\varpi_1, \varpi_2 \cdots, \varpi_n$.

Let

$$Q_n = I - P_n \qquad (3.26)$$

Then we have

$$\|Au\| = \left\|(-\Delta)^m u\right\| \ge \lambda_{n+1}\|u\|, \forall u \in Q_n(H^{2m} \cap H_0^m), \qquad (3.27)$$

$$\|Q_n u\| \le \|u\|, u \in H. \qquad (3.28)$$

Lemma 3.3 For any $U_0, V_0 \in B$, Let

$$Q_{n_0}(t) = Q_{n_0}(U(t) - V(t)) = Q_{n_0}W(t) = (w_{n_0}, z_{n_0})^T,$$

then

$$\left\|W_{n_0}(t)\right\|^2 \le e^{-2k_1 t}\left\|U_0 - V_0\right\|_{V_1}^2. \qquad (3.29)$$

Proof Taking Q_{N_0} in (3.16), we receive

$$\frac{1}{2}\frac{d}{dt}\left\|W_{n_0}(t)\right\|^2 + \left(Q_{n_0}\left(H(U) - H(V)\right), W_{n_0}(t)\right) = 0. \qquad (3.30)$$

Which is

$$\frac{d}{dt}\left\|W_{n_0}(t)\right\|^2 + 2k_1\left\|W_{n_0}(t)\right\|_{V_1}^2 \le 0. \qquad (3.31)$$

By using Gronwall inequality, we obtain

$$\left\|W_{n_0}(t)\right\|^2 \le e^{-2k_1 t}\left\|U_0 - V_0\right\|_{V_1}^2. \qquad (3.32)$$

Lemma 3.4 (Discrete squeezing property). For any $U_0, V_0 \in B$, if

$$\| P_{n_0}(S(T^*)U_0 - S(T^*)V_0)\|_{V_1} \le \|(I - P_{n_0})(S(T^*)U_0 - S(T^*)V_0)\|_{V_1}, \qquad (3.33)$$

then

$$\| S(T^*)U_0 - S(T^*)V_0\|_{V_1} \le \frac{1}{8}\|(U_0 - V_0)\|_{V_1}. \qquad (3.34)$$

Proof If $\| P_{n_0}(S(T^*)U_0 - S(T^*)V_0)\|_{V_1} \le \|(I - P_{n_0})(S(T^*)U_0 - S(T^*)V_0)\|_{V_1}$, then

$$\begin{aligned} &\left\|S(T^*)U_0 - S(T^*)V_0\right\|_{V_1}^2 \\ &\le \left\|(I - P_{n_0})(S(T^*)U_0 - S(T^*)V_0)\right\|_{V_1}^2 + \left\|P_{n_0}(S(T^*)U_0 - S(T^*)V_0)\right\|_{V_1}^2 \qquad (3.35) \\ &\le 2\left\|(I - P_{n_0})(S(T^*)U_0 - S(T^*)V_0)\right\|_{V_1}^2 \\ &\le 2e^{-2k_1 T^*}\left\|U_0 - V_0\right\|_{V_1}^2. \end{aligned}$$

Let T^* be large enough,

$$e^{-2k_1T^*} \le \frac{1}{128}. \tag{3.36}$$

Combining with (3.35),(3.36), we receive

$$\| S(T^*)U_0 - S(T^*)V_0\|_{V_1}^2 \le \frac{1}{64}\| U_0 - V_0\|_{V_1}^2. \tag{3.37}$$

Then

$$\| S(T^*)U_0 - S(T^*)V_0\|_{V_1} \le \frac{1}{8}\|(U_0 - V_0)\|_{V_1}. \tag{3.38}$$

Theorem 3.2 Let $(H_1)-(H_3)$ be in force, assume that $f \in H$, $(u_0, v_0) \in V_k$, $k = 1, 2$,then the semigroup $S(t)$ determined by (1.1)-(1.3) possesses an $(V_2, V_1)-$exponential attractor on B,

$$\mathrm{M} = \bigcup_{0\le t\le T^*} S(t)(\mathrm{A} \cup \left(\bigcup_{j=1}^{\infty}\bigcup_{k=1}^{\infty} S(T^*)^j (E^{(k)})\right)) \tag{3.39}$$

Proof According to Theorem 2.1, Lemma3.2 and Lemma 3.4, Theorem 3.2 is easily proven.

IV. Conclusions

The paper's main results deal with exponential attractors. In section 2, we introduce some basic concepts. In section 3, we prove the squeezing property of the nonlinear semigroup associated with this equation, then the existence of existence of exponential attractor is proved.

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