Generating of new types of fractal structures and developing of formula for the effective thermal conductivity of fractal porous media

Fethi Kamışlı, Ercan Aydoğmuş

Abstract— **Developing formula or numerical method for the different types of fractal structures of porous material is area of interest. The porous media is modelled by different new fractal structures such as T, H, L types, the rectangle type, the hollow square type of fractal structure and Sierpinski carpet type of fractal structure available in the literature. In the present investigation the formula for computing effective thermal conductivity for the different types of fractal structures used to model fractal porous structures was developed as a function of porosity, the thermal conductivities of a solid and a fluid. Furthermore, two dimensional heat conduction equation with variable thermal conductivities was solved to compute the effective thermal conductivity as a function of porosity, the thermal conductivities of a solid and a fluid in the different types of fractal structures. The effective thermal conductivities of fractal porous media calculated using the experimental, parallel, serial and the developed formulas were compared one another for each fractal structure to evaluate validity of developed formula and numerical calculation. The effective thermal conductivities for each type of fractal structure were depicted in a graph as a function of porosity. Moreover, the method of finite difference was used to solve heat conduction in a fractal porous material and thus, the effective thermal conductivities of different types of fractal structures. The developed formula and the numerical calculation for computing thermal conductivities agree well with the available formulas in the literature.**

Index Terms— **Effective thermal conductivity, modeling fractal structure, porosity.**

I. INTRODUCTION

The porous materials are used in all areas of engineering. The heat conduction in porous materials is subject of many investigations since the most of isolation materials, soils, catalysts, many materials subjected to dry and even a packed bed reactor can be taken into account as porous materials. Heat conduction in a porous material is quite complicated since the effective thermal conductivity is affected by properties of a solid matrix, the size and spatial distribution of pores, the type, component and state of fluid in pores, the pressure and temperature [1].

The different types of fractal structure have been used to model the porous materials to develop a model for some types of transport problems and the heat conduction in a porous media is one of them [2,3,4].

Haui et al. [5] examined heat conduction in porous media in an effort to calculate the effective thermal conductivity numerically in the different types of fractal structures. The finite volume element was used in the computing of the effective thermal conductivity of porous material in a shape of rectangular as functions of thermal conductivities of a solid and a fluid and porosity. Collishaw and Evans [6] investigated the porosity in the solid matrix and the microstructure of geometry to calculate the effective thermal conductivity of the porous solid. In the study by Fu et al. [7] the effective thermal conductivity of cellular ceramics was calculated using different models. The effective thermal conductivity of porous material filled with a fluid was calculated as a function of porosity, the thermal conductivities of a solid and a fluid. Singh et al. [8] studied on determining the effective thermal conductivity of porous material in the shape of a three dimensional cube. It was assumed that at the first stage all pores were in the shape of a cube and at the second stage all pores were in the shape of a sphere. In order to estimate the effective thermal conductivity of two-phase materials accurately, Samantray et al. [9] compared the formulas available in the literature. The effective thermal conductivity of two-phase materials was calculated as functions of porosity and thermal conductivity of each phase. For evaluating effective thermal conductivity of porous media the existing formulas were derived based on the assumption that the porous consists of only either two parallel layers or two serial layers. Considering the porous media consisting of the combined two parallel layers and two serial layers is more realistic than that of only either two parallel or two serial layers. Therefore, in this paper the formula for the effective thermal conductivity was developed based on the porous media consisting of the combined two parallel layers and two serial layers. Thereby, the superiority of the developed formula over the existing formulas is unquestionable. The new formula was theoretically developed as functions of the porosity, the thermal conductivities of a solid and a fluid. Furthermore, the porous media was modelled in terms of entirely new fractal structures such as T, H, L types, the hollow square type, the rectangle type of fractal structures generated based on fractal theory and the heat conduction in these types of fractal structures was simulated using the method of finite difference. Those fractal structures affect the porosity of the media. The effective thermal conductivity for each fractal structure was computed using the developed formula and the numerical simulation which give more accurate results than the parallel and serial formulas available in the literature.

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II. FORMULAS FOR COMPUTING AN EFFECTIVE THERMAL CONDUCTIVITY

The porosity (ε) for the different types of fractal structures is computed using following equation.

$$\varepsilon = 1 - \left(a / b^2\right)^N \qquad (1)$$

N stands for the number of construction stages, *a* ratio of solid in the fractal structure, b^2 unit area of porous material. The effective thermal conductivity of a porous material can be calculated using following equations.

$$k_{eff} = k_f{}^{\varepsilon} k_s{}^{(1-\varepsilon)} \qquad (2)$$

$$k_{efp} = \varepsilon k_f + (1-\varepsilon) k_s \qquad (3)$$

$$k_{efs} = 1 / \left[\varepsilon / k_f + (1-\varepsilon) / k_s\right] \qquad (4)$$

In these equations k_s, k_f and ε denote thermal conductivities of a solid, a fluid and porosity, respectively. k_{eff}, k_{efp} and k_{efs} stand for the effective thermal conductivities calculated from the experimental, parallel and serial formulas, respectively.

Eq. (2) called the experimental formula, available in the literature, was obtained by fitting experimental data. Eq. (3) and Eq. (4) called the parallel and serial formulas, available in the literature also, were derived theoretically.

A. *Examining of Parallel and Serial Formula for Effective Thermal Conductivity*

The total rate of heat transfer through the composite wall consisting of two parallel layers can be expressed as follows:

$$q = \left(\frac{k_1 A_1}{\ell} + \frac{k_2 A_2}{\ell}\right)(T_1 - T_2) \qquad (5)$$

In this equation ℓ stands for thickness of the parallel wall and its length is constant, A_1 and A_2 denote the areas of layer-1 and layer-2 perpendicular to heat flow and these areas for porous media can be expressed as a function of porosity; $A_1 = (1-\varepsilon) A$, $A_2 = \varepsilon A$ if the layer-1 and layer-2 are taken to be as a solid and a fluid part and *A* represents total heat transfer area. The total rate of heat transfer through the parallel composite wall in terms of effective thermal conductivity can be expressed as follows:

$$q = k_{efp} A \frac{(T_1 - T_2)}{\ell} \qquad (6)$$

Substituting A_1 and A_2 and Eq. (6) into Eq. (5) gives following equation.

$$k_{efp} = \varepsilon k_f + (1-\varepsilon) k_s$$

This equation known as parallel formula for evaluating the effective thermal conductivity of porous media in the literature is equal to Eq. (3).

Similarly the total rate of heat transfer through the composite wall consisting of two serial layers can be expressed as follows:

$$q = \frac{(T_1 - T_2)}{[\ell_1 / k_1 A + \ell_2 / k_2 A]} \qquad (7)$$

Here ℓ_1 and ℓ_2 denote thickness of layer-1 and layer-2 in serial arrangement and for porous media they can be expressed as a function of porosity as: $\ell_1 = (1-\varepsilon)\ell$ and $\ell_2 = \varepsilon \ell$ if the layer-1 and layer-2 are taken to be as a solid and a fluid part and ℓ represents total length of the composite wall.

Substituting ℓ_1, ℓ_2 and Eq. (6) into Eq. (7) results in following equation.

$$k_{efs} = 1 / \left[\varepsilon / k_f + (1-\varepsilon) / k_s\right]$$

This equation known as serial formula for defining effective thermal conductivity of porous media in the literature is equal to Eq. (4).

B. *Derivation of Formula for Effective Thermal Conductivity for Porous Media Consisting of the Combined Parallel–Serial Arrangement*

By considering the available formulas in the literature, we developed Eq. (9) theoretically in the present study for computing an effective thermal conductivity of a porous media as functions of the thermal conductivities of a solid and a fluid and porosity (ε).

Fig. 1 illustrates the composite wall consisting of serial and parallel layers.

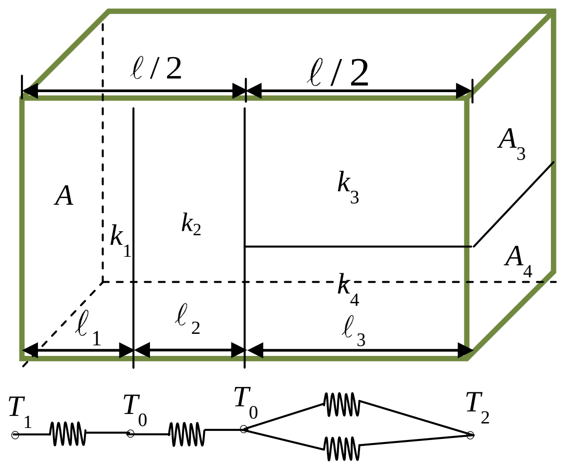

Fig. 1. The composite wall consisting of two serial and two parallel layers.

The total rate of heat transfer through the composite wall consisting of two serial and two parallel layers can be expressed as follows:

$$q = \frac{(T_1 - T_2)}{\left[\frac{\ell_1}{k_1 A} + \frac{\ell_2}{k_2 A} + \frac{\ell_3}{k_3 A_3 + k_4 A_4}\right]} \qquad (8)$$

In Eq. (8) A, A_3 and A_4 denote the heat transfer area of layer-1 or layer-2, layer-3 and layer-4, respectively and for porous media they can be expressed $A_3 = (1 - \varepsilon)\, A$, $A_4 = \varepsilon\, A$ if the layer-1 and layer-3 are taken to be as a solid part and layer-2 and layer-4 are taken to be a fluid part and A represents total heat transfer area. ℓ_1, ℓ_2 and ℓ_3 denote thickness of layer-1, layer-2 and layer-3 or layer-4 can be expressed as a function of porosity as: $\ell_1 = (1-\varepsilon)\ell/2$, $\ell_2 = \varepsilon\ell/2$ and $\ell_3 = \ell/2$.

Substituting ℓ_1, ℓ_2, ℓ_3 A_3, A_4 and Eq. (6) into Eq. (8) results in following equation.

$$k = \frac{2}{\left[\frac{\varepsilon}{k_f} + \frac{(1-\varepsilon)}{k_s}\right] + \frac{1}{\varepsilon k_f + (1-\varepsilon) k_s}} \qquad (9)$$

Eq. (9) developed in the present paper can be used to evaluate the effective thermal conductivity. As can be seen in the derivation Eq. (9) is developed by considering the porous media consisting of the combined two serial and two parallel layers while deriving of Eq. (3) and Eq. (4) considering only two parallel layers and two serial layers, respectively. Therefore, it is expected that the developed formula should give more accurate results and those of Eq. (3) and Eq. (4) since the porous media is assumed to be consisted of the combined parallel–serial arrangement instead of assuming only either parallel or serial arrangement.

Later in this paper it will be proved that the developed formula gives more accurate results than the parallel and serial formulas since the data obtained from the developed formula layout between those obtained from parallel and serial formulas and are closer to those obtained from the experimental formula.

III. GENERATION OF DIFFERENT TYPE FRACTAL STRUCTURES OF A POROUS MEDIA

In order to understand how to generate square structure type we take up the generation of the Sierpinski carpet type structure. The square is one of the most well-known alike mathematical clumps. Its initial form is a white Sierpinski carpet and a black square is put into the exact center of the white one (see Fig. 2a). The generator is going to be formed by removing black square and remaining white ones. In the next iteration stage, each of remaining sub- square is again subdivided to obtain new sub-squares (see Fig. 2a).

The generated Sierpinski carpet (Si), the H type, T type and the hollow square type (S) fractal structures are illustrated in Fig. 2. They are generated based on the fractal generator. As can be seen in Fig.1, the generator is a square divided into $a \times a$ congruent sub-squares, b of these are black and the other $a^2 - b$ are white. As mentioned previously while the white sub-squares remain, black squares are removed. In the next iteration stage, each of the remaining sub-square is again subdivided into $a \times a$ congruent sub-squares and the pattern of the generator is repeated on it. Although the procedure for generating square type fractal structure is given here, the same procedure is followed to generate the T type, H type and the hollow square type fractal structures. The dimension and porosity for each fractal structure can be calculated as follows:

Sierpinski carpet (Si) dimension:
$D_{Si} = \ln(8) / \ln(3) = 1.893$, porosity: $\varepsilon_{Si} = 1 - (8/9)^N$
T fractal structure dimension:
$D_T = \ln(20) / \ln(5) = 1.861$, porosity: $\varepsilon_T = 1 - (20/25)^N$
H fractal structure dimension:
$D_H = \ln(18) / \ln(5) = 1.796$, porosity: $\varepsilon_H = 1 - (18/25)^N$
Hollow square fractal structure dimension:
$D_S = \ln(17) / \ln(5) = 1.760$, porosity: $\varepsilon_S = 1 - (17/25)^N$

IV. RESULTS AND DISCUSSION

Sierpinski carpet type, T type, H type and hollow square type structure based on fractal theory are generated to model the porous media, and heat conduction in these structures is simulated using the methods of finite difference and finite element (ANSYS). Effective thermal conductivity of the porous media as functions of the porosity, thermal conductivities of the solid and the fluid was examined. In this study, the thermal conductivity of solid and the thermal conductivity of fluid in pores were taken to be 10 W/m K and 2 W/m K, respectively. Fig. 3 illustrates the variation of the effective thermal conductivity of the porous media as functions of the porosity, the thermal conductivities of the solid and the fluid in the Sierpinski carpet type structure. The values of the effective thermal conductivity of porous media calculated by using different formulas are shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6 for the Sierpinski carpet type, the T type, the H type and the hollow square type fractal structures, respectively. As can be seen in Fig. 3 the effective thermal conductivity of the porous media decreases with increasing the porosity since amount of fluid increases with increasing the porosity of the media and the identical trends can be observed in all types of fractal structures. As a result, the effective thermal conductivity decrease with increasing the porosity when the thermal conductivity of a fluid is less than that of a solid. In other words, this situation happens when the thermal conductivity of a solid is larger than that of a fluid. If the thermal conductivity of a fluid is taken to be higher than that of a solid, the opposite trend will be observed since the amount of a fluid in pores increases with the increasing the porosity of media.

In other words, the effective thermal conductivity is going to increase with increasing the amount of a substance having higher thermal conductivity or is going to decrease with

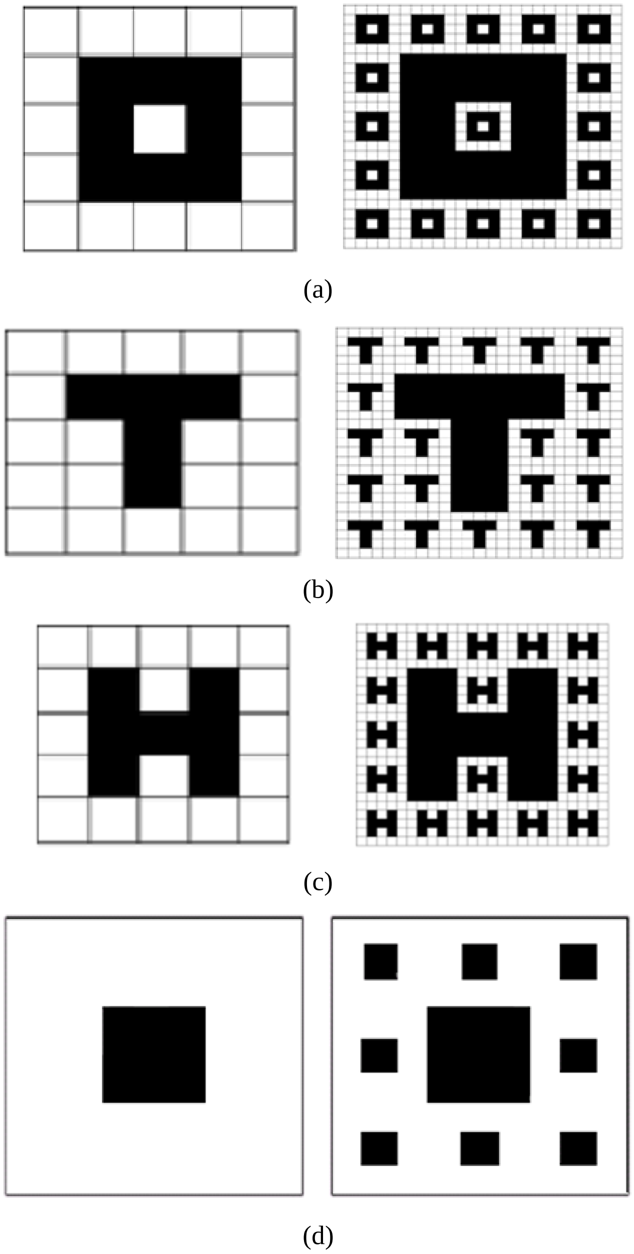

Fig. 2. Sierpinski carpet type (a), T type (b), H type (c) and hollow square type (d) fractal structures.

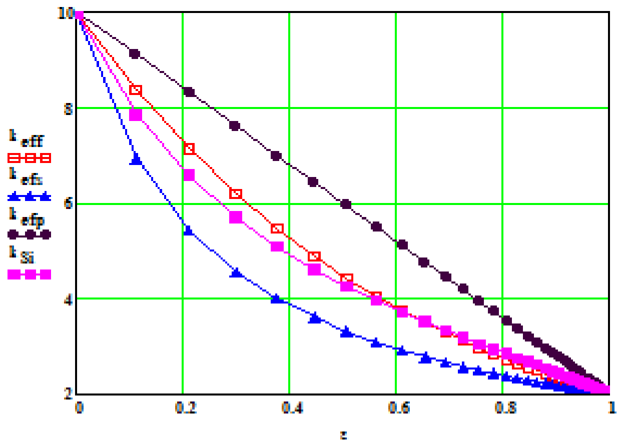

Fig. 3. The comparison of effective thermal conductivities calculated from different formulas as a function of the porosity in the Sierpinski carpet type fractal structure.

increasing the amount of a substance having lower thermal conductivity. As expected the effective thermal conductivity of porous media starts to change from a value of thermal conductivity of a solid to a value of thermal conductivity of a fluid with increasing porosity from 0 to 1. This trend is independent of a formula used to calculate the effective thermal conductivity as a function of porosity as seen in Fig. 3, Fig. 4, Fig. 5 and Fig. 6. The effective thermal conductivity calculated from the parallel formula decreases linearly from the value of thermal conductivity of the solid to that of thermal conductivity of the fluid. On the other hand, the effective thermal conductivity calculated by using other formulas such as the serial, the experimental and the developed formula in the present study decreases asymptotically by increasing the porosity of media (see Fig. 3, Fig. 4, Fig. 5 and Fig. 6). As can be seen in Fig. 4 the results evaluated from the developed formula layout between those obtained from the parallel and serial formulas and agree well with those obtained from the experimental formula. This indicates that the developed formula gives more accurate results than those of the parallel and the serial formula (see Fig. 3 – Fig. 6). The profile of the thermal conductivities calculated from the developed formula is closer to that of the experimental formula than those of the parallel and the serial formulas. When Fig. 3 is compared with Fig. 4- Fig. 6, the Sierpinski carpet gives smoother results than the T type, the H type and the hollow square type fractal structures. It can also be said that spatial distribution of pores has significant effect on the effective thermal conductivity of porous media since the Sierpinski carpet fractal structure has different the spatial distribution of pores from those of the T type, the H type and the hollow square type fractal structures. Porosity increases with increasing the number of construction stages (N) in all types of fractal structures. However, an increase in the porosity with N can be put in order as follows: $\varepsilon_S > \varepsilon_H > \varepsilon_T > \varepsilon_{Si}$ (see Fig. 3 – Fig. 6). The porosity of media at the same number of construction stage can be aligned as above that is $\varepsilon_S > \varepsilon_H > \varepsilon_T > \varepsilon_{Si}$. Therefore, each fractal structure of a porous media gives slightly different results in terms of the effective thermal conductivity at the same porosity (compare Fig. 3 – Fig. 6).

As can be seen in Fig. 3 – Fig. 6, the effective thermal conductivity of the porous media decreases faster in the serial formula than in other formulas at the low porosity while a decrease in the effective thermal conductivity is slower in the parallel formula than in the other formulas at the same porosity. The same trend is observed in all types of the fractal structures.

Fig. 7 illustrates that the comparison of several types of fractal structures in terms of the effective thermal conductivities calculated from the developed formula in the present investigation. These fractals were generated to model structure of the porous media. As can be seen in Fig. 7 the Sierpinski carpet has the largest values of the effective thermal conductivity as a function of porosity at all values of the porosity, which the Sierpinski carpet type fractal structure has the lowest porosity when compared with the other fractal structures. As mentioned earlier, the effective thermal conductivity starts to change from the thermal conductivity of the solid to that of the fluid when the porosity varies from 0 to 1.

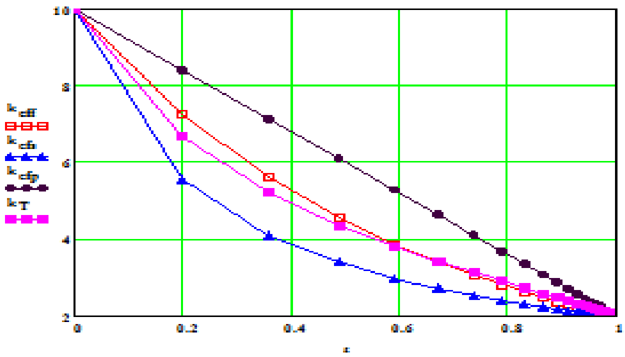

Fig. 4. The comparison of effective thermal conductivities calculated from different formulas as a function of the porosity in the T type fractal structure.

Although the rectangle type and L type fractal structures are not shown here, these fractal structures were examined in the frame of this study. As can be seen in Fig. 7 the rectangle type fractal structure has the highest porosity since the lowest values of effective thermal conductivity is encountered in this fractal structure. This figure indicates that the rectangle fractal structure (D) has the highest increase in porosity with increasing the number of construction stages (*N*). On the other hand, the Sierpinski carpet type fractal structure has the lowest increase in the porosity with increasing the number of construction stages (*N*). The order of an increase in porosity as a function of *N* can be expressed as follows:

$$\varepsilon_D > \varepsilon_K > \varepsilon_H > \varepsilon_L = \varepsilon_T > \varepsilon_S \, .$$

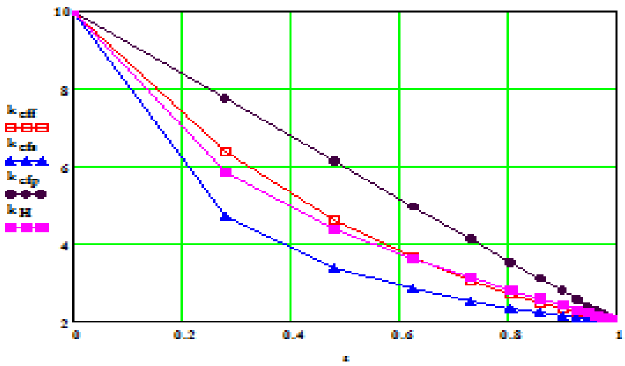

Fig. 5. The comparison of effective thermal conductivities calculated from different formulas as a function of the porosity in the H type fractal structure.

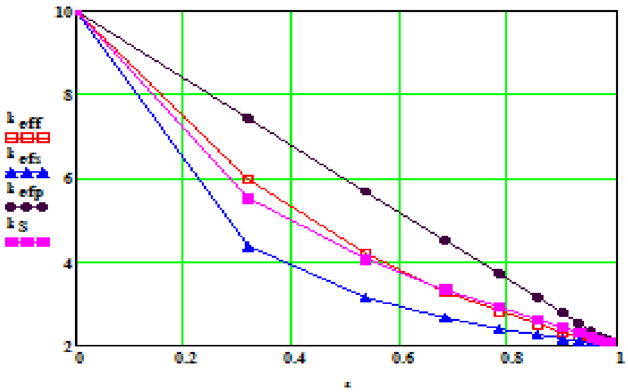

Fig. **6**. The comparison of effective thermal conductivities computed from different formulas as a function of the porosity in the hollow square fractal structure.

Here D, S, H, L, T and Si denote the rectangle type, the hollow square type, the H, L, T type fractal structures and the Sierpinski carpet type, respectively. Since the effective thermal conductivity of porous media depends strongly on the porosity, the thermal conductivities of a solid and a fluid, the effective thermal conductivity increases/decreases according to the fraction of substance having a higher/lower thermal conductivity increases in a porous media. As can be seen in Fig. 7, the effective thermal conductivity of the porous media at the same porosity can be put in order from a high value to a low value as follows:

$$k_{Si} > k_T = k_L > k_H > k_S > k_D$$

In order to determine temperatures at each grid point in the considered fractal structure, each fractal structure is conformed to grid points. The grid points and boundary conditions for the T type fractal structure, as an example, is shown in Fig. 8.

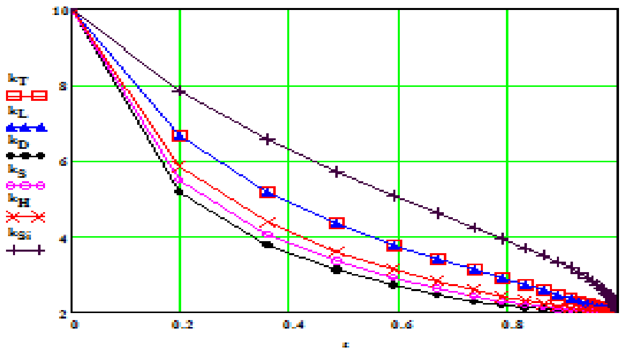

Fig. 7. The comparison of fractal structures in terms of the effective thermal conductivity.

In determining temperature distribution in the fractal structure the thermal conductivities of the solid, the fluid and the interface were taken to be 10, 2 and 6 W/m K, respectively. Δx and Δy were taken to be equal to one another. Two dimensional heat conduction equation in Cartesian coordinate system in terms of the central finite difference can be written as in Eq. (10).

In Eq. (10) k_i denotes thermal conductivity of a substance which can be the solid, the fluid or the interface depending on position in the fractal structure. Δx and Δy stand for increment in x and y direction, respectively and $T_{i,j}$ represents temperature at *i,j* point in the fractal structure.

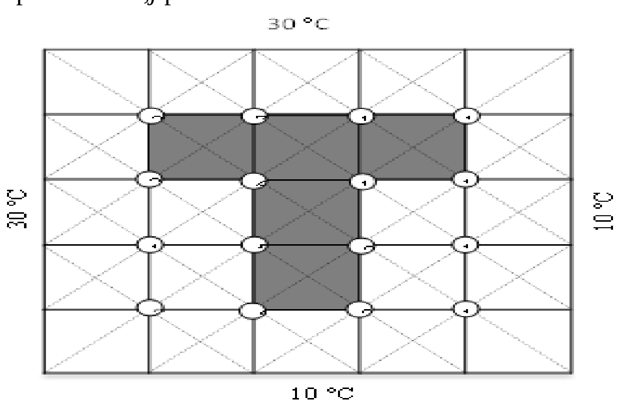

Fig. 8. The grid points and boundary conditions for the T type fractal structure.

$$T_{i,j}=\frac{\left[k_1T_{i-1,j}+k_2T_{i+1,j}+k_3T_{i,j-1}+k_4T_{i,j+1}\right]}{\left[k_1+k_2+k_3+k_4\right]} \qquad (10)$$

In order to compute the effective thermal conductivity of porous media modeled as a T fractal structure, the heat conduction was computed using an average value of high and low temperatures of the eight points and an average value of heat conduction was also computed using these eight points.

Several types of fractals are generated to model the structures of porous media and the heat conduction in these structures is simulated using the finite difference method. In order to determine the average value of temperature at each point for each stage (see Fig. 8), eight neighbor points were used in the considered fractal structure. For the T fractal structure there are 36 grid points at the first stage and 25×36 grid points at the second stage and etc. (see Fig. 2). During determining of the average temperature at a point the thermal conductivities of the solid, the fluid and the interface were used according to the position of point considered. The average value of temperature was obtained from the following equation. The following boundary conditions were used to determine temperature distribution and thus conduction heat transfer and the effective thermal conductivity. As shown in the boundary conditions the maximum and minimum temperatures are 30 °C and 10 °C (see Fig. 8) and therefore, the maximum temperature difference is 20 °C.

$$T(0,y)=30\ ^{o}\mathrm{C}$$
$$T(\ell_1,y)=10\ ^{o}\mathrm{C}$$
$$T(x,0)=10\ ^{o}\mathrm{C}$$
$$T(x,\ell_2)=30\ ^{o}\mathrm{C}$$

$$T_{i,j}=\frac{\left[k_1T_{i-1,j}+k_2T_{i+1,j}+k_3T_{i,j-1}+k_4T_{i,j+1}+k_5T_{i-1,j-1}+k_6T_{i-1,j+1}+k_7T_{i+1,j-1}+k_8T_{i+1,j+1}\right]}{\left[k_1+k_2+k_3+k_4+k_5+k_6+k_7+k_8\right]} \qquad (11)$$

The total value of heat conduction was obtained by adding heat conduction one another for the considered points and the average value of heat conduction was obtained by dividing the total heat conduction with the number of points. Therefore, the effective thermal conductivity of porous media can be expressed as follows:

$$k_{ef}=\left[\left(\sum_{i=1}^{n}q_i\right)\frac{1}{n}\right]/\left(T_{\max}-T_{\min}\right) \qquad (12)$$

Here $T_{\max}$ and $T_{\min}$ stand for the maximum and minimum temperatures at two neighbor points in the fractal structure and each of them is the average temperature of eight adjoining points. The heat conduction is taken place from a point at high temperature to an adjoining point at relatively low temperature.

The porous media with boundary condition is shown in Fig. 8. As can be seen in Fig. 8 the temperatures of sides remain to be constant while the temperature varies inside porous media since the temperatures of the sides of porous media were kept at the constant conditions during entire computation.

The effective thermal conductivities obtained from the different numerical methods and the formulas are illustrated in Fig. 9. As can be seen in Fig. 9, the effective thermal conductivities, a function of porosity of media, calculated from the developed formula (Eq. (9)) and the experimental formula (Eq. (2)) and computed from the method of finite difference agree well with one another. Here the experimental formula taken from the related literature can be used as a reference one to evaluate our results although the experimental formula can contain some uncertainty and inaccuracy since it has been obtained by fitting experimental data. As can be seen in Fig. 9 the values for the effective thermal conductivities at each porosity value are in agreement with each other. The agreement between the values obtained using the developed formula and computed from the finite difference method and the values obtained from experimental formula indicates that the formula we developed gives the reliable results. The agreement between the results obtained from the method of the finite difference and the experimental formula points out that the method of the finite element works well in terms of computing the effective thermal conductivities also.

When the thermal conductivity of the slab is assumed to be constant, two-dimensional heat conduction equation can be written as follows:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \qquad (13)$$

First Eq. (13) was expressed in terms of the dimensionless temperature and then analytically solved by the aid of the method of separation variables subjected to the following boundary conditions:

$$\phi(x,y)=\left(T(x,y)-T_c\right)/\left(T_h-T_c\right)$$
$$\phi(0,y)=1$$
$$\phi(\ell_1,y)=0$$
$$\phi(x,0)=0$$
$$\phi(x,\ell_2)=1$$

The general solution of Eq. (13) with the dimensionless temperature was obtained as follows:

$$\phi(x,y)=\left(C_1\cos\lambda x+C_2\cos\lambda x\right)\left(C_3e^{\lambda y}+C_4e^{-\lambda y}\right) \qquad (14)$$

After determining integral constants namely C_1, C_2, C_3 and C_4, Eq. (14) can be given as follows:

$$\phi(x,y)=\frac{T(x,y)-T_c}{T_h-T_c}=-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\cos\left[(n-1/2)\pi x/\ell_1\right]\frac{\sinh\left[(n-1/2)\pi y/\ell_1\right]}{\sinh\left[(n-1/2)\pi\ell_2/\ell_1\right]} \qquad (15)$$

When the numerical values of T_h and T_c are taken to be equal to 30 °C and 10 °C, respectively, to obtain temperature profiles as function of x and y. As can be seen from Eq. (15) the temperature in the porous slab with constant thermal conductivity varies with x and y smoothly from the lowest temperature (10 °C) to the highest temperature (30 °C).

The temperature profile in the porous slab having a variable thermal conductivity was obtained by solving following two dimensional heat conduction equation by using ANSYS.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=0 \qquad (16)$$

The temperature profiles for the T type, the H type and the hollow square fractal structure are given in Fig. 11 – Fig. 12. Although temperature profiles for other fractal structure such as the L, D and Si were not shown here, they were also computed. ANSYS was used to obtain the temperature distribution and conduction heat transfer in the porous slab and thus, the effective thermal conductivity in the different types of fractal structures at the just first stage. When it is assumed that the porous media can be modelled in the form of T type fractal structure, the temperature distribution in this structure will be in Fig. 11. As can be seen in the figure the temperature profile in a substance having a high thermal conductivity changes more rapidly than that in a substance having a relatively low thermal conductivity as expected since the heat conduction is directly proportional with thermal conductivity.

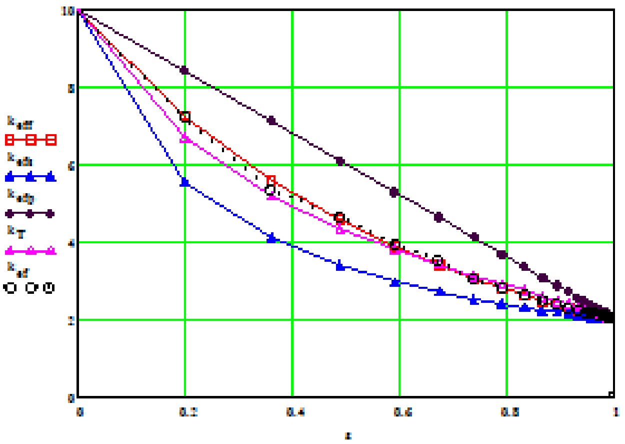

Fig. 9. Comparison of the effective thermal conductivities obtained using different formulas and the method of finite difference in the T type fractal structure.

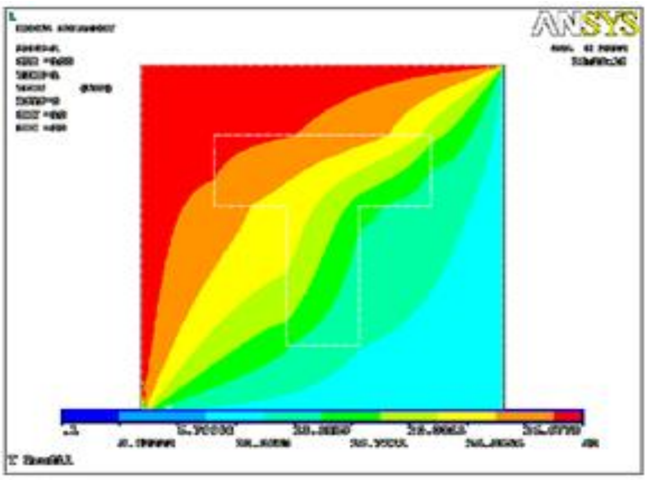

Fig. 10. Temperature distribution in the T type fractal structure obtained from ANSYS.

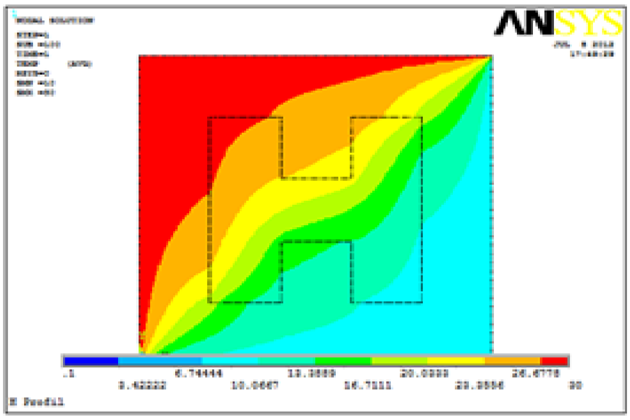

Fig. 11. Temperature distribution in the H type fractal structure calculated from ANSYS.

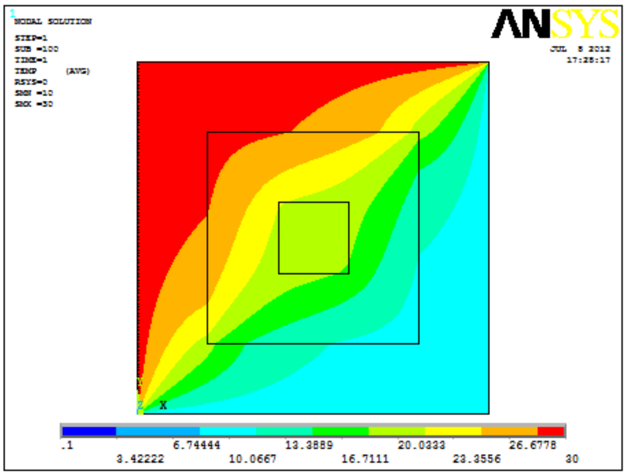

Fig. 12. Temperature distribution in the hollow square type fractal structure obtained from ANSYS

V. CONCLUSIONS

The porous media was modelled to be in the form of T, H, the Hollow Square, L, D and Sierpinski carpet types fractal structures. The formula theoretically developed here was used to compute the effective thermal conductivity as functions of the porosity, the thermal conductivities of a solid and a fluid. The developed formula gives more accurate results than the parallel and serial formulas available in the literature since the data obtained from this equation layout between those obtained from the parallel and serial formulas without depending on fractal structure types. It is found out that the results obtained from the developed formula are closer to those obtained from the experimental formula than those obtained from the parallel and serial formulas at each porosity. The similar trend was observed for all types of the fractal structures considered here. The effective thermal conductivity was also numerically computed as functions of the porosity, thermal conductivities of a solid and a fluid. In numerical computation, after determining the temperatures at each grid point by the method of finite difference, the average value of adjoining eight grid point temperatures was used to determine the temperature at one grid point. It was observed that the results computed from the numerical solution were also closer to those obtained from the experimental formula than those obtained from the parallel and serial formulas. As a result, the effective thermal conductivities computed using the developed formula and the numerical solution are closer to those obtained from the experimental formula than those obtained from the parallel and serial formulas at each porosity for all types of fractal structures.

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