Review of soft sensors for position control of the PMSM Rotating frame *vs* Stationary frame

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Abstract— **The main objective of this paper is to provide a simple analysis of the various sensorless control methods used to estimate the rotor position of a permanent-magnet synchronous machine. In particular model-based soft sensors are presented. The model used can be expressed either in a reference frame related to stator or in a reference frame related to rotor. The advantages and the drawbacks of each method are highlighted.**

Index Terms— **Permanent magnet synchronous motor, position estimation, review, rotor, sensorless drives.**

I. Introduction

Permanent magnet synchronous machines (PMSM) are widely used in industrial applications because of their special advantages, such as high efficiency, high power/torque density, and a wide constant power zone. To achieve high performance field-oriented control, accurate rotor position information, which is usually measured by rotary encoders is necessary. However, the cost of a sensor may exceed the cost of a small motor in some applications. Also, the presence of the mechanical sensors not only increases the cost and complexity of the total material with additional wiring but also reduces its reliability with additional sensitivity to external disturbances. In addition, it may be difficult to install and maintain a position sensor due to the limited space and rigid work environment with high vibration or high temperature. Therefore, the idea is to replace the mechanical sensor by a soft sensor which offers a number of attractive properties one of them being a low cost alternative to hardware speed measurement used in classical motor drives [1–3].

Therefore the position sensor is replaced by a model-based soft sensor where the position estimation can be correctly obtained from the standard models of the permanent-magnets synchronous machines. This model can be expressed either in a reference frame fixed to stator windings: the fixed reference *(α - β)* or in a reference frame fixed to rotor windings: the rotating frame *(d - q)*. For the first instance, the *(α - β)* model allows direct estimation of the rotor position. The speed is deduced by using a Phase-Locked Loop (PLL) which avoids the direct derivation of the position [4–6]. For the second case, the *(d - q)* model can be obtained by applying the Park transform to the *(α - β)* model. However, this transform requires the position measurement which is not available in the case of sensorless applications. With this model the velocity estimate precedes the estimate of the position which can be derived by integrating the estimated speed [7].

The aim of this paper is to present a review about conventional methods and current trends in sensorless control of permanent magnet synchronous motor drive. At the end an observation on the convergence of the *(d - q)* soft sensors is presented.

This paper is organized as follows:

- First, the motor equations for the PMSM in different coordinates are illustrated.
- Second, a classification for different sensorless control methods for the PMSM is given.
- Third, the *(d - q)* and the *(α - β)* soft sensors are described.

II. Synchronous Machine Modeling

Nowadays, synchronous machines, especially PMSM spread increasingly like actuators in automated industries where they replace the DC motors. Indeed, the use of the permanent magnets synchronous machines had a very important development in several industrial sectors because of its simplicity of design, its high-speed operating ability, performance in terms of torque-to-weight ratio and low maintenance.

The PMSM considered in this study has a stator composed of a three-phase winding represented by the three axes (a, b, c) phase-shifted with respect to one another by an electrical angle of 120° and a rotor having *p* pole pairs. To simplify the modeling of the machine, we adopt the usual simplifying assumptions given in the majority of references [8]

- The stator windings are symmetrical and have perfect sinusoidal distribution along the air gap.
- The permanence of the magnetic paths on the rotor is independent of the rotor positions.
- Saturation and hysteresis effects are inexistent.

In this section, the model of PMSM is presented in three different frames: in the three-phase frame *(abc)*, in the stationary reference frame *(α - β)* and in the rotating frame *(d - q)*. The different coordinates are defined in Fig. 1, which shows the model of a PMSM.

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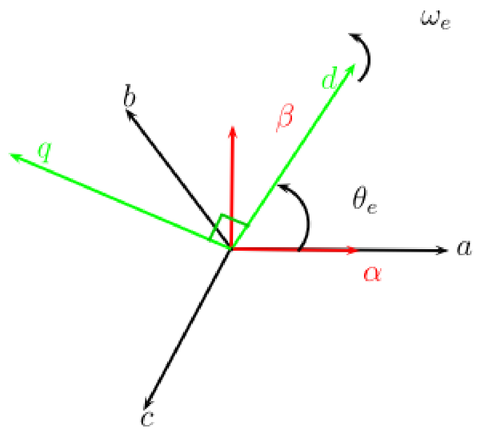

Fig. 1. Three-phase / Stationary / Rotating frame.

A. *Model in the three-phase frame*

In the three-phase frame *(abc)*, the stator voltages of PMSM can be expressed by

$$[u_s]=[R_s][i_s]+\frac{d}{dt}[\psi_s] \tag{1}$$

where $[u_s]=[u_a\ u_b\ u_c]^T$ are the phase voltages, $[i_s]=[i_a\ i_b\ i_c]^T$ are the phase currents, $[\psi_s]=[\ \psi_a\ \psi_b\ \psi_c]^T$ are the phase flux linkages and $[R_s]$ is the phase resistance given by

$$[R_s]=\begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

Equation (1) presents a nonlinear model of the PMSM which is difficult to use. Thus, different changes and transformations are considered in order to reduce the complexity of this system.

B. *Model in the stationary reference frame*

In order to model the three-phase system previously introduced through a two-phase model, Clarke transformation, given by (3), is used.

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}=\frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{3}$$

The unsaturated PMSM can be modeled in the stationary reference frame *(α - β)* by the following set of equations [9]

$$L\frac{di_{\alpha\beta}}{dt}=-R_s i_{\alpha\beta}+\omega_e\psi_{pm}\begin{bmatrix} \sin\theta_e \\ -\cos\theta_e \end{bmatrix}+u_{\alpha\beta} \tag{4}$$

$$T_e=\frac{3p}{4}\psi_{pm}\left(i_\alpha\cos\theta_e-i_\beta\sin\theta_e\right) \tag{5}$$

where

$i_{\alpha\beta}=[i_\alpha\ i_\beta]^T$ is the stator current vector, $u_{\alpha\beta}=[u_\alpha\ u_\beta]^T$ is the motor terminal voltage vector, R_s is the stator windings resistance, L is the cyclic inductance, ψ_{pm} is the magnetic flux, p is the number of pole pairs, θ_e is the electrical rotor position and ω_e is the electrical rotor speed. Throughout this paper, we shall assume that the mechanical parameters, position and speed, are unknown.

C. *Model in the rotating frame*

This model is obtained by implementing Park's transform to (4) and (5). Then, PMSM can be given by

$$L\frac{di_d}{dt}=-R_s i_d+u_d+\omega_e L i_q,$$
$$L\frac{di_q}{dt}=-R_s i_q+u_q-\omega_e\left(L i_d+\psi_{pm}\right), \tag{6}$$
$$T_e=p\psi_{pm}i_q.$$

In this type of model, the delivered torque T_e is proportional to the quadrature current i_q. Thereby, this representation is the most frequently used in the field-oriented control of PMSM. However, Park's transform requires the use of the electrical position which is not available in sensorless applications. Several techniques have been developed to estimate, in a first step, the velocity from which we can deduce the position by integration.

III. PROPOSED SOFT SENSORS

During the years, researchers have developed many sensorless techniques. These methods can be classified under two main categories according to the speed range of the machine:

- Methods adequate for standstill and low speed region (High-frequency soft sensors),
- Methods suitable for rated and high speed region (Model-based soft sensors).

In the first category, position estimation is based on the anisotropy of the magnetic circuit and it can be obtained from high-frequency signal injection. These techniques are usually applied to the interior permanent magnet synchronous motor (IPMSM) as it involves the effects of salience.

As for the second category, this estimate can be correctly obtained from the standard models of synchronous and asynchronous machines which give excellent results. The model used can be expressed either in the stationary reference frame *(α, β)* or in the rotating frame *(d, q)*.

Several techniques inspired from control theory [10]–[12], such as adaptive observers [13]–[15], reference models [16]–[18], extended Kalman filter [19] and reduced-order observers [20]–[21].

A more detailed classification of proposed soft sensors can be presented as follows:

1. High-frequency soft sensors
   - 1.1. Pulses injection,
   - 1.2. Signal injection,
   - 1.3. INFORM method.
2. Model-based soft sensors
   - 2.1. Back-EMF estimation
     - 2.1.1. Currents interruption,
     - 2.1.2. Zero crossings,
     - 2.1.3. Voltage integration,
     - 2.1.4. Back-EMF calculation.
   - 2.2. Estimators and observers

2.2.1. Sliding mode observers,
2.2.2. Reduced-order observers,
2.2.3. Kalman filter,
2.2.4. Adaptive observers,
2.2.5. Model reference adaptive system (MRAS) observers.

In the next sections of this paper, we present some model-based soft sensors expressed in the stationary reference frame *(α, β)* and in the rotating frame *(d, q)*. First, we present a sliding mode and a nonlinear observer developed in the stationary reference frame. Second, a reduced-order and a MRAS observer expressed in the rotating frame are exposed. Finally, some observations on the convergence of the model-based soft sensors is illustrated.

IV. Soft sensors expressed in the Stationary frame

For synchronous machines, such approaches allow a direct estimate of the position without using the speed estimation. These methods are based on the estimation of the electromotive forces which are proportional to the rotor speed.

A. Sliding mode observer

Sliding mode control is a powerful robust nonlinear control technique that has been intensively developed during the last years. This technique have been approved as one of the main approaches for the design of robust observers for complex nonlinear dynamic systems operating under conditions of uncertainty. Sliding mode observer (SMO) are receiving a very important attention because of its robustness, speed of convergence, good performance and innate insensitivity to parameter variations and disturbances. It takes advantage of the error between the measured and the estimated stator currents to obtain the back-EMF which contains the rotor position and speed information. However, the chattering phenomenon is the major disadvantage of SMO. The chattering creates system oscillations, performance degradation and even instability to the system. The effect of chattering is predominant in conventional SMO where a signum function is used as control law [22]–[23].

In order to reduce the effect of chattering and to increase the accuracy of the estimation of the rotor position at low speed and the stability of the system at high-speed, a saturation function is used in such systems [24]–[26]. Using this function greatly reduces the chattering, but the presence of a low-pass filter can not be avoided.

Another solution is to replace the signum function by the sigmoid function [27]–[29]. Significant reduction in chattering is noticed in the system with saturation and sigmoid function compared to the conventional signum function. The sigmoid function could even reduce chattering far better than saturation function. Use of sigmoid function also avoids the use of low pass filter and hence the compensation algorithms.

The sliding mode control design is composed of three steps. First, a sliding surface, given by S(x)=0, should be designed. Then, the convergence condition must be define. Finally, the control law have to be determined.

The electrical equations of the PMSM in the stationary reference frame may be formulated as follows:

$$\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} = \frac{1}{L_q}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} - \frac{R_s}{L_q}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \frac{1}{L_q}\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} \quad (7)$$

e_α and e_β are the extended electromotive force.

$$e_\alpha = -\left(2L_1\omega_e i_d + \omega_e \psi_{pm}\right)\sin\theta_e \quad (8)$$

$$e_\beta = \left(2L_1\omega_e i_d + \omega_e \psi_{pm}\right)\cos\theta_e \quad (9)$$

where $L_1 = \dfrac{L_d - L_q}{2}$.

A sliding mode observer can be defined as follows [30]:

$$\begin{bmatrix} \dot{\hat{i}}_\alpha \\ \dot{\hat{i}}_\beta \end{bmatrix} = \frac{1}{L_q}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} - \frac{R_s}{L_q}\begin{bmatrix} \hat{i}_\alpha \\ \hat{i}_\beta \end{bmatrix} - \frac{K_{SM}}{L_q}\begin{bmatrix} sign\left(\hat{i}_\alpha - i_\alpha\right) \\ sign\left(\hat{i}_\beta - i_\beta\right) \end{bmatrix} \quad (10)$$

where K_{SM} is the gain of the observer and

$$sign\left(\hat{i}_\alpha - i_\alpha\right) = \begin{cases} 1 & if \left(\hat{i}_\alpha - i_\alpha\right) > 0 \\ 0 & if \left(\hat{i}_\alpha - i_\alpha\right) = 0 \\ -1 & if \left(\hat{i}_\alpha - i_\alpha\right) < 0 \end{cases} \quad (11)$$

Subtracting the model equation (7) from (10) yields the error dynamics along the sliding surface

$$\begin{bmatrix} \dot{\tilde{i}}_\alpha \\ \dot{\tilde{i}}_\beta \end{bmatrix} = -\frac{R_s}{L_q}\begin{bmatrix} \tilde{i}_\alpha \\ \tilde{i}_\beta \end{bmatrix} + \frac{1}{L_q}\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} - \frac{K_{SM}}{L_q}\begin{bmatrix} sign\left(\tilde{i}_\alpha\right) \\ sign\left(\tilde{i}_\beta\right) \end{bmatrix} \quad (12)$$

where $\tilde{i}_\alpha$ and $\tilde{i}_\beta$ are the observation errors given by

$$\tilde{i}_\alpha = \hat{i}_\alpha - i_\alpha \quad (13)$$

$$\tilde{i}_\beta = \hat{i}_\beta - i_\beta \quad (14)$$

Let us select the Lyapunov function as

$$V = \frac{1}{2}\left(\tilde{i}_\alpha^2 + \tilde{i}_\beta^2\right) \quad (15)$$

Then

$$\dot{V} = -\frac{R_s}{L_q}\left(\tilde{i}_\alpha^2 + \tilde{i}_\beta^2\right) + \frac{1}{L_q}\left(e_\alpha\tilde{i}_\alpha + e_\beta\tilde{i}_\beta\right) - \frac{K_{SM}}{L_q}\left(\left|\tilde{i}_\alpha\right| + \left|\tilde{i}_\beta\right|\right) \quad (16)$$

Equation (16) shows that, if K_{SM} is large enough, i.e.,

$$K_{SM} > \max\left\{\left|e_\alpha\right|, \left|e_\beta\right|\right\} \quad (17)$$

then $\dot{V} < 0$ is always guaranteed until $\tilde{i}_\alpha = 0$ and $\tilde{i}_\beta = 0$.

In order to force the convergence of the observed current values to the measured current values, the desired error values between the observed and actual current values in the stationary reference frame are set to zero. Then we apply the control scheme presented in Fig. 2. Once the sliding surface is reached and the observation error tends to zero, the extended electromotive force can be given by the following equations:

$$e_\alpha = K_{SM}\, sign_{eq}\left(\tilde{i}_\alpha\right) \quad (18)$$

$$e_\beta = K_{SM}\, sign_{eq}\left(\tilde{i}_\beta\right) \quad (19)$$

Low-pass filters are used to extract e_α and e_β. Finally, the rotor position is obtained by

$$\hat{\theta}_e = -\tan^{-1}\left(\frac{e_\alpha}{e_\beta}\right) = -\tan^{-1}\left(\frac{K_{SM}\, sign_{eq}\left(\tilde{i}_\alpha\right)}{K_{SM}\, sign_{eq}\left(\tilde{i}_\beta\right)}\right) \quad (20)$$

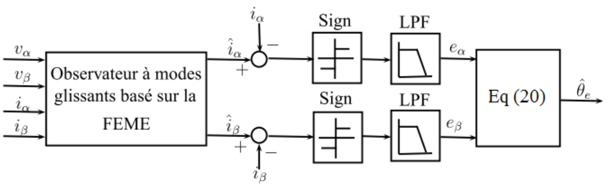

Fig. 2. Block diagram of the sliding mode observer.

B. Nonlinear observer

The presented technique is based on the model of the PMSM given by (4) and (5) and on an online reconstruction of the back-electromotive force. Based on the following equation

$$\Psi_{\alpha\beta} = Li_{\alpha\beta} + \psi_{pm}\begin{bmatrix}\cos\theta_e \\ \sin\theta_e\end{bmatrix} \quad (21)$$

we can obtain the rotor position θ_e:

$$\theta_e = a\tan 2\left(\Psi_\beta - Li_\beta, \Psi_\alpha - Li_\alpha\right) \quad (22)$$

But in the sensorless control, the position is un known so we cannot compute the actual value of the total flux. As a solution, we can use the estimated value of the flux given by the following equations:

$$\dot{\hat{\Psi}}_\alpha = -R_s i_\alpha + u_\alpha + v\eta_\alpha\left(\psi_{pm}^2 - \eta_\alpha^2 - \eta_\beta^2\right) \quad (23)$$

$$\dot{\hat{\Psi}}_\beta = -R_s i_\beta + u_\beta + v\eta_\beta\left(\psi_{pm}^2 - \eta_\alpha^2 - \eta_\beta^2\right) \quad (24)$$

where ν is a positive scalar that establishes the convergence speed of the observer and

$$\eta_\alpha = \hat{\Psi}_\alpha - Li_\alpha \quad (25)$$

$$\eta_\beta = \hat{\Psi}_\beta - Li_\beta \quad (26)$$

The estimated flux $\hat{\Psi}_\alpha$ and $\hat{\Psi}_\beta$ are available for measurement as it only depends on the currents (i_α, i_β) and the voltages (u_α, u_β). Then the estimated rotor position can be given by:

$$\hat{\theta}_e = a\tan 2\left(\hat{\Psi}_\beta - Li_\beta, \hat{\Psi}_\alpha - Li_\alpha\right) \quad (27)$$

As is ill-advised to obtain the speed estimate through numerical differentiation of the position estimates. The speed observer can be given by the following equations [9].

$$\begin{aligned}\hat{z}_1 &= K_p\left(\hat{\theta}_e - z_1\right) + K_i z_2 \\ \hat{z}_2 &= \hat{\theta}_e - z_1 \\ \hat{\omega}_e &= K_p\left(\hat{\theta}_e - z_1\right) + K_i z_2\end{aligned} \quad (28)$$

where K_i and K_p are proportional and integral gain of the controller, respectively.

V. Soft sensors expressed in the rotating frame

Several soft sensors expressed in the rotating frame have been developed. In this case, the velocity estimate precedes the estimate of the position which can be derived by integrating the estimated speed.

A. Reduced-order Luenberger observer

The state-space representation of the PMSM model can be written as [31]:

$$\begin{cases}\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\omega}_e\end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \frac{L_q}{L_d}\omega_e & 0 \\ -\frac{L_d}{L_q}\omega_e & -\frac{R_s}{L_q} & -\frac{\psi_{pm}}{L_q} \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega_e\end{bmatrix} + \begin{bmatrix}\frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0\end{bmatrix}\begin{bmatrix}u_d \\ u_q\end{bmatrix} \\ \omega_e = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega_e\end{bmatrix}\end{cases} \quad (29)$$

To simplify the writing, we will use the following notations:

$$a_d = \frac{R_s}{L_d}, a_q = \frac{R_s}{L_q}, b = \frac{\psi_{pm}}{L_q}, c_d = \frac{L_q}{L_d}, c_q = \frac{L_d}{L_q}, l_d = \frac{1}{L_d}, l_q = \frac{1}{L_q}$$

Based on the model of PMSM given by (29), we can write the state space representation of the machine follows:

$$\begin{cases}\dot{x} = \begin{bmatrix}-a_d & c_d\omega_e & 0 \\ -c_q\omega_e & -a_q & -b \\ 0 & 0 & 0\end{bmatrix}x + \begin{bmatrix}l_d & 0 \\ 0 & l_q \\ 0 & 0\end{bmatrix}u \\ y = \begin{bmatrix}0 & 0 & 1\end{bmatrix}x\end{cases} \quad (30)$$

In this model, ω_e plays both the role of a state variable and a parameter appearing in the dynamical matrix. However, in the case of a sensorless control, the speed is not measured and only the currents i_d and i_q are accessible to measurement. Hence, the state space representation can be given by the following equation

$$\begin{cases}\dot{\breve{x}} &= \breve{A}\breve{x} + \breve{B}u \\ y_{mes} &= \breve{C}x + D_{mes}u\end{cases} \quad (31)$$

where

$$\breve{x} = x = \begin{bmatrix}x_1 & x_2\end{bmatrix},\ y_{mes} = x_1,\ \breve{A} = \begin{bmatrix}\breve{A}_{11} & \breve{A}_{12} \\ \breve{A}_{21} & \breve{A}_{22}\end{bmatrix},$$

$$\breve{B} = \begin{bmatrix}\breve{B}_1 & \breve{B}_2\end{bmatrix}, C_{mes} = \breve{C} = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0\end{bmatrix}, D_{mes} = \breve{D} = 0$$

The state vector $\breve{x}$ in (31) can be divided into two sub-vectors: $\breve{x}_1 = x_1$ and $\breve{x}_2 = x_2 = \omega_e$.

So

$$\breve{A}_{11} = \begin{bmatrix}-a_d & c_d\omega_e \\ -c_q\omega_e & -a_q\end{bmatrix}, \breve{A}_{12} = \begin{bmatrix}0 \\ -b\end{bmatrix}, \breve{B}_1 = \begin{bmatrix}l_d & 0 \\ 0 & l_q\end{bmatrix}$$

$$\breve{A}_{21} = \begin{bmatrix}0 & 0\end{bmatrix}, \breve{A}_{22} = 0, \breve{B}_2 = 0$$

Therefore, it is necessary to observe $\breve{x} = \begin{bmatrix}\breve{x}_1^T & \breve{x}_2^T\end{bmatrix}^T$ with $\breve{x}_1 \in R^m$, $\breve{x}_2 \in R^q$ and $q = n - m$. In general, we must reconstruct

$$\chi = Sx = S_1x_1 + S_2x_2 = S\breve{M}\breve{x} = \breve{S}_1\breve{x}_1 + \breve{S}_2\breve{x}_2 \quad (32)$$

where

$$S\breve{M} = \breve{S} = \begin{bmatrix}\breve{S}_1 & \breve{S}_2\end{bmatrix} \quad (33)$$

Equation (31) leads to

$$\begin{cases} \dot{\breve{x}}_2 = \breve{A}_{22}\breve{x}_2 + \begin{bmatrix} \breve{A}_{21} & \breve{B}_2 \end{bmatrix} \begin{bmatrix} \breve{x}_1 \\ u \end{bmatrix} \\ \dot{\breve{x}}_1 = \breve{A}_{12}\breve{x}_2 + \begin{bmatrix} \breve{A}_{11} & \breve{B}_1 \end{bmatrix} \begin{bmatrix} \breve{x}_1 \\ u \end{bmatrix} \end{cases} \quad (34)$$

where

$$\breve{A}_{22} = \breve{A}, \quad \begin{bmatrix} \breve{A}_{21} & \breve{B}_2 \end{bmatrix} = \breve{B}, \quad \breve{A}_{12} = \breve{C},$$

$$\begin{bmatrix} \breve{A}_{11} & \breve{B}_1 \end{bmatrix} = \breve{D}, \quad \dot{\breve{x}}_1 = \breve{y}, \quad \begin{bmatrix} \breve{x}_1 \\ u \end{bmatrix} = \breve{u}.$$

The Luenberger observer has the following structure:

$$\begin{cases} \dot{z} = \breve{F}z + \breve{P}\breve{y} + \breve{R}\breve{u} \\ \hat{\chi} = \breve{L}z + \breve{Q}\breve{y} + \breve{G}\breve{u} \end{cases} \quad (35)$$

where $z \in R^q$. The observability of $\left(\breve{A}, \breve{C}\right) = \left(\breve{A}_{22}, \breve{A}_{12}\right)$ is a necessary and sufficient condition for the existence of this observer. If $\left(\breve{A}, \breve{C}\right)$ is observable then so is $\left(\breve{A}_{22}, \breve{A}_{12}\right)$. Indeed

$$rank\begin{bmatrix} \breve{C} \\ \left(\lambda I - \breve{A}\right) \end{bmatrix} = rank\begin{bmatrix} I_m & 0 \\ \left(\lambda I_m - \breve{A}_{11}\right) & \breve{A}_{12} \\ \breve{A}_{21} & \left(\lambda I_q - \breve{A}_{22}\right) \end{bmatrix} = n \ \forall \lambda \in C$$

$$\Rightarrow rank\begin{bmatrix} \breve{A}_{12} \\ \left(\lambda I_q - \breve{A}_{22}\right) \end{bmatrix} = q \ \forall \lambda \in C$$

Luenberger's idea is to design the observer so as to satisfy $z = T\hat{\breve{x}}$ with $\hat{\breve{x}} = \begin{bmatrix} \breve{x}_1^T & \hat{\breve{x}}_2^T \end{bmatrix}^T$ is a reconstruction of $\breve{x}$ composed of $\breve{x}_1$ and $\hat{\breve{x}}_2$: ($\hat{\breve{x}}_2$ a reconstruction of $\breve{x}_2$). The matrix T can be divided as follows:

$$T = \begin{bmatrix} Z & \breve{T} \end{bmatrix} \quad (36)$$

such that

$$z = \breve{T}\hat{\breve{x}}_2 + Z\breve{x}_1 = \breve{T}\breve{x}_2 + Z\breve{x}_1 + \mu$$

where

$$\mu = \breve{T}\varepsilon_2 = \breve{T}\left(\hat{\breve{x}}_2 - \breve{x}_2\right)$$

So, we have

$$\dot{\mu} = \breve{F}\mu + \left(\breve{F}\breve{T} - \breve{T}\breve{A} + \breve{P}\breve{C} - Z\breve{C}\right)\breve{x}_2 + \left(\breve{R} + \breve{P}\breve{D} - \breve{T}\breve{B} - Z\breve{D} + \breve{F}\begin{bmatrix} Z & 0 \end{bmatrix}\right)\breve{u} \quad (37)$$

To ensure the asymptotic convergence of z to Tx we must satisfy the following constraints

$$\begin{cases} \breve{F} \text{ is stable}, \\ \breve{F}\breve{T} - \breve{T}\breve{A} + \left(\breve{P} - Z\right)\breve{C} = \breve{F}\breve{T} - \breve{T}\breve{A}_{22} + \left(\breve{P} - Z\right)\breve{A}_{12} = 0, \\ \breve{R} + \left(\breve{P} - Z\right)\breve{D} - \breve{T}\breve{B} + \begin{bmatrix} Z & 0 \end{bmatrix} = 0. \end{cases} \quad (38)$$

Under these constraints, μ tends to 0 and thus $\breve{T}\hat{\breve{x}}_2$ tends to $\breve{T}\breve{x}_2$. The third equation in (9) can be rewritten as:

$$\begin{cases} \breve{R}_1 - \breve{T}\breve{A}_{12} + \left(\breve{P} - Z\right)\breve{A}_{11} + \breve{F}Z = 0 \\ \breve{R}_2 - \breve{T}\breve{B}_2 + \left(\breve{P} - Z\right)\breve{B}_1 = 0 \end{cases} \quad (39)$$

where $\breve{R} = \begin{bmatrix} \breve{R}_1 & \breve{R}_2 \end{bmatrix}$.

Moreover, the real goal is to rebuild χ. The static equation in (35) leads to

$$\hat{\chi} = \breve{L}\breve{T}\breve{x}_2 + \breve{L}Z\breve{x}_1 + \breve{L}\mu + \breve{Q}\breve{C}\breve{x}_2 + \breve{Q}\breve{D}\breve{u} + \breve{G}\breve{u}$$

$$\Leftrightarrow \hat{\chi} = \left(\breve{Q}\breve{C} + \breve{L}\breve{T}\right)\breve{x}_2 + \left(\begin{bmatrix} \breve{L}Z & 0 \end{bmatrix} + \breve{Q}\breve{D} + \breve{G}\right)\breve{u} + \breve{L}\mu$$

To ensure an efficient reconstruction in the steady state such that $\lim_{t\to\infty} \mu(t) = 0$, we must ensure that

$$\lim_{t\to\infty}\left(\left(\breve{Q}\breve{C} + \breve{L}\breve{T}\right)\breve{x}_2 + \left(\begin{bmatrix} \breve{L}Z & 0 \end{bmatrix} + \breve{Q}\breve{D} + \breve{G}\right)\breve{u}\right) = \breve{S}_1\breve{x}_1 + \breve{S}_2\breve{x}_2$$

Let $\breve{G} = \begin{bmatrix} \breve{G}_1 & \breve{G}_2 \end{bmatrix}$, then we have

$$\begin{cases} \breve{Q}\breve{C} + \breve{L}\breve{T} = \breve{S}_2 \\ \begin{bmatrix} \breve{L}Z & 0 \end{bmatrix} + \breve{Q}\breve{D} + \breve{G} = 0 \end{cases}$$

In other terms, we must satisfy the following constraints:

$$\begin{cases} \breve{Q}\breve{A}_{12} + \breve{L}\breve{T} = \breve{S}_2 \\ \breve{L}Z + \breve{Q}\breve{A}_{11} + \breve{G}_1 = \breve{S}_1 \\ \breve{Q}\breve{B}_1 + \breve{G}_2 = 0 \end{cases} \quad (40)$$

These constraints, given by equation (38) and equation (40) may be difficult to verify both from computational and practical point of views, so we set the following arbitrary choice: $\breve{T} = I_q$, $\breve{P} = 0$ and $\breve{Q} = 0$.

So $z = Z\breve{x}_1 + \hat{\breve{x}}_2$ and

$$\begin{cases} \breve{F} = \breve{A} + Z\breve{C} = \breve{A}_{22} + Z\breve{A}_{12} \\ \breve{R} = \begin{bmatrix} \breve{A}_{21} + Z\breve{A}_{11} - \left(\breve{A}_{22} + Z\breve{A}_{12}\right)Z & \breve{B}_2 + Z\breve{B}_1 \end{bmatrix} \\ \breve{L} = \breve{S}_2 \\ \breve{G} = \begin{bmatrix} \breve{S}_1 - \breve{S}_2 Z & 0 \end{bmatrix} \end{cases}$$

The Luenberger observer is then given by

$$\begin{cases} \dot{z} = Fz + Py_{mes} + Ru \\ \hat{\chi} = Lz + Qy_{mes} + Gu \end{cases} \quad (41)$$

where

$$\begin{cases} F = \breve{F} = \breve{A}_{22} + Z\breve{A}_{12} \\ P = \breve{R}_1 = \breve{A}_{21} + Z\breve{A}_{11} - \left(\breve{A}_{22} + Z\breve{A}_{12}\right)Z \\ R = \breve{R}_2 - \breve{R}_1\breve{D} = \breve{B}_2 + Z\breve{B}_1 - \left(\breve{A}_{21} + Z\breve{A}_{11} - \left(\breve{A}_{22} + Z\breve{A}_{12}\right)Z\right)\breve{D} \\ L = \breve{L} = \breve{S}_2 \\ Q = \breve{G}_1 = \breve{S}_1 - \breve{S}_2 Z \\ G = -\breve{G}_1\breve{D} = \left(\breve{S}_2 Z - \breve{S}_1\right)\breve{D} \end{cases} \quad (42)$$

It comes then to determine a reduced observer by assuming that $\chi = \omega_e$ which leads to $\breve{S}_1 = \begin{bmatrix} 0 & 0 \end{bmatrix}$ and $\breve{S}_2 = 1$. It should be noted that $\left(\breve{A}_{22}, \breve{A}_{12}\right)$ is observable. Let Z be expressed as $Z = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}$. Based on the previous result, we have

$$\begin{cases} F = -\dfrac{\alpha_2 \psi_{pm}}{L_q} \\ P = \begin{bmatrix} -\alpha_1 a_d - \alpha_2 c_q \omega_e + \alpha_1 \alpha_2 b & \alpha_1 c_d \omega_e - \alpha_2 a_q + \alpha_2^2 b \end{bmatrix} \\ R = \begin{bmatrix} \alpha_1 l_d & \alpha_2 l_q \end{bmatrix} \\ L = 1 \\ Q = \begin{bmatrix} -\alpha_1 & -\alpha_2 \end{bmatrix} \\ G = 0 \end{cases} \quad (43)$$

From the equation (9), we obtain

$$\begin{cases} \dot{z} = -\dfrac{\alpha_2 \psi_{pm}}{L_q} z + \left(-\alpha_1 \dfrac{R_s}{L_d} - \alpha_2 \dfrac{L_d}{L_q}\omega_e + \alpha_1 \alpha_2 \dfrac{\psi_{pm}}{L_q}\right) i_d + \dfrac{\alpha_1}{L_d} u_d \\ \qquad + \left(\alpha_1 \dfrac{L_q}{L_d}\omega_e - \alpha_2 \dfrac{R_s}{L_q} + \alpha_2^2 \dfrac{\psi_{pm}}{L_q}\right) i_q + \dfrac{\alpha_2}{L_q} u_q \\ \hat{\omega}_e = z - \alpha_1 i_d - \alpha_2 i_q \end{cases} \quad (44)$$

In this particular observer, the reconstruction gap of the speed $\tilde{\omega}_e = \hat{\omega}_e - \omega_e$ is described by the following equation

$$\dot{\tilde{\omega}}_e = -\alpha_2 \frac{\psi_{pm}}{L_q} \tilde{\omega}_e = F\tilde{\omega}_e \quad (45)$$

Such a dynamic is characterized by only one pole: F.

Hence α_2 must be chosen positive for asymptotic of F and α_1 can be chosen equal to zero as it has no effect on this pole. This choice is due to the structure of the matrix $\breve{A}_{12}$ that present a null component. Therefore, α_1 can be taken zero which simplifies the expression of the observer.

In this analysis, the nonlinearity of the model is hidden by the fact that ω_e is also a parameter of the dynamic matrix. But this nonlinearity appears suddenly and causes a problem. The calculated observer depends on ω_e which is not measured. To implement it, we must replace, ω_e by $\hat{\omega}_e$ in the model of the observer, that is a reasonable approximation since the observer has converged. For this, we must choose the gain α_2 such as the bandwidth of the observer is both greater than the bandwidth of the speed controller and smaller than the current controller ($|\alpha_\omega| << |\alpha_O| < |\alpha_C|$). Hence, we obtain:

$$\begin{cases} \dot{z} = \dfrac{-\alpha_2 \psi_{pm}}{L_q} z - \alpha_2 \dfrac{L_d}{L_q} \hat{\omega}_e i_d + \left(-\alpha_2 \dfrac{R_s}{L_q} + \alpha_2^2 \dfrac{\psi_{pm}}{L_q}\right) i_q + \dfrac{\alpha_2}{L_q} u_q \\ \hat{\omega}_e = z - \alpha_2 i_q \end{cases} \quad (46)$$

The rotor position can be obtained by integration of the rotor speed:

$$\hat{\theta}_e(t) = \int_0^t \hat{\omega}_e(\tau)\, d\tau \quad (47)$$

B. MRAS observer

This approach consists in using two different models: a reference model and an adaptive model. In [32], Three MRAS estimators are designed to estimate the rotor speed, the rotor-flux magnitude, and the stator resistance. It is demonstrated that the simultaneous estimation of the stator resistance and the rotor-flux magnitude is not possible, so two separate estimation schemes are proposed. The first scheme given by Fig. 3 can estimate the speed of the machine and the stator resistance while the rotor flux magnitude is set to its nominal value. The adaptive error signal equations of the rotor speed and the stator resistance are chosen as follows

$$\varepsilon_\omega = -\frac{1}{a_2 \hat{\psi}_r} \Delta i_{sy} \quad (48)$$

$$\varepsilon_{a_1} = \frac{1}{\hat{i}_s^2}\left(-\hat{i}_{sy}\, \Delta i_{sx} + \hat{i}_{sx} \Delta i_{sy}\right) \quad (49)$$

where

$$a_1 = \frac{R_s}{L_s},\ a_2 = \frac{1}{L_s},\ G_{c\omega} = K_{p\omega} + \frac{K_{i\omega}}{s},\ G_{ca_1} = K_{pa_1} + \frac{K_{ia_1}}{s}$$

The second scheme given by Fig. 4 is used to estimate the speed and magnetic flux where the stator resistance is set at its nominal value. Adaptive error signals are given by the following equations

$$\varepsilon_\omega = -\frac{1}{a_2 \hat{\psi}_r} \Delta i_{sy} \quad (50)$$

$$\varepsilon_{\psi_r} = -\frac{1}{a_2 \omega_r^2} \Delta i_{sx} \quad (51)$$

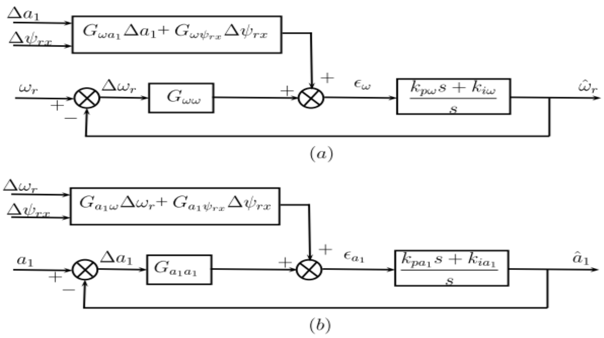

Fig. 3. Scheme 1: (a) Rotor-speed estimator. (b) Stator resistance estimator.

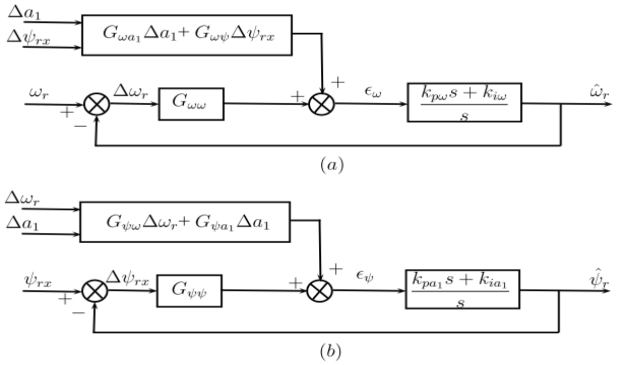

Fig. 4. Scheme 2: (a) Rotor-speed estimator. (b) Rotor flux estimator.

VI. Observations on the Convergence of the Rotating Soft Sensors

Assumption: Let $\tilde{\theta}_{e0}$ the initial rotor position estimation error. Then, a soft sensor expressed in the rotating frame maintains this error in open loop and eliminates it in closed loop.

This assumption has be proved by different simulation tests. Fig. 5 and Fig. 6 show the open-loop and the closed loop simulation results of two rotating soft sensors presented above. In order to prove that this is a characteristic of *(d - q)* soft sensors, we present in Fig.7 the simulation results achieved in open loop and closed loop of the nonlinear observer which is expressed in the stationary frame. The initial position estimation error is equal to $\frac{\pi}{4}$.

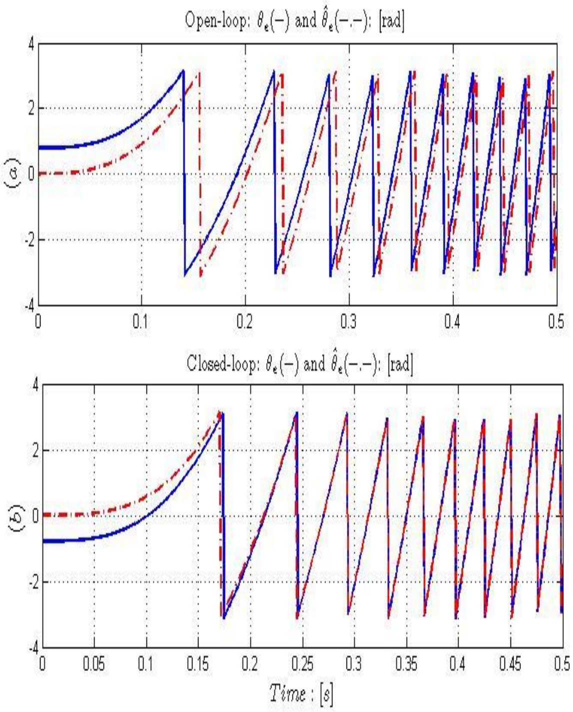

Fig. 5. MRAS observer. (a) Open-loop simulation (b) Closed-loop simulation

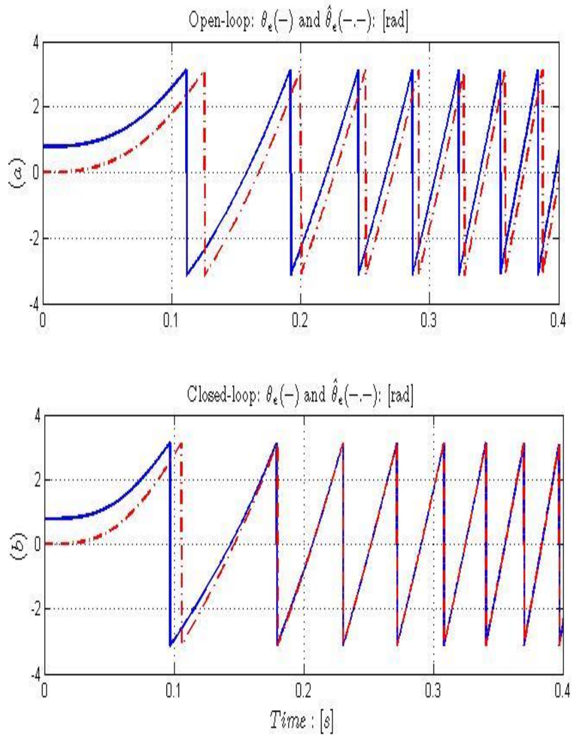

Fig. 6. Reduced-order observer. (a) Open-loop simulation (b) Closed-loop simulation

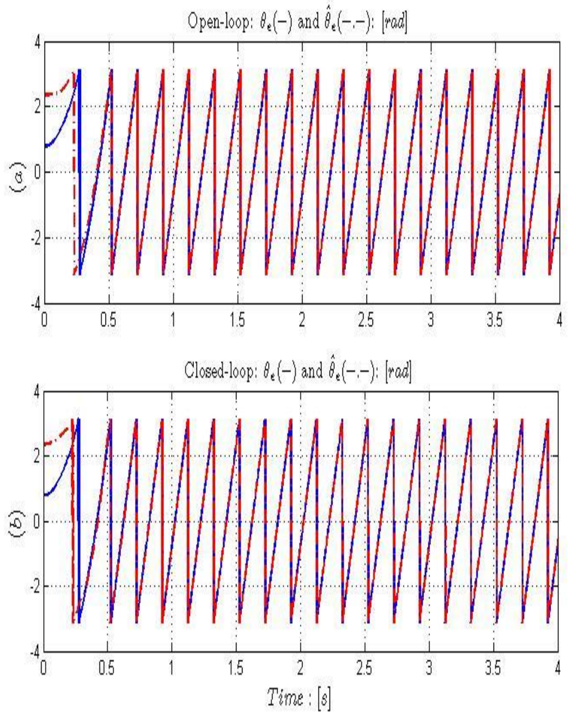

Fig. 7. Nonlinear observer. (a) Open-loop simulation (b) Closed-loop simulation

VII. Conclusion

In this paper, we have presented a review in state of the art techniques for sensorless position estimation of permanent magnet synchronous motor drives . In particular, the *(d - q)* and the *(α - β)* soft sensors have been described. At the end, some observations on the convergence of the model-based soft sensors have been illustrated.

REFERENCES

[1] R. Abdelli, D. Rekioua, T. Rekioua (2011), "Performances improvements and torque ripple minimization for VSI fed induction machine with direct control torque". *ISA Trans*, Vol. 50, Iss. 2, pp. 213-219.

[2] A.Y. Achour, B.Mendil, S. Bacha, I. Munteanu (2009), "Passivity-based current controller design for a permanent-magnet synchronous motor". *ISA Trans*, Vol. 48, Iss. 3, pp. 336-346.

[3] B. Zhang, Y. Pi, Y. Luo (2012), "Fractional order sliding-mode control based on parameters auto-tuning for velocity control of permanent magnet synchronous motor". *ISA Trans*, Vol. 51, Iss. 5, pp. 649-656.

[4] M. Hinkkanen, T. Tuovinen, L. Harnefors, J. Luomi (2010), "Analysis and design of a position observer with stator-resistance adaptation for PMSM drives". International Conference on Electrical Machines ICEM, Rome, Italy, pp. 1-6.

[5] A. Piippo, M. Hinkkanen, J. Luomi (2009), "Adaptation of motor parameters in sensorless PMSM drives". IEEE *Trans. Ind. Appl*, Vol. 45, Iss. 1, pp. 203-212.

[6] X. Zhang, L. Sun, K. Zhao, L. Sun (2013), "Nonlinear speed control for PMSM system using sliding-mode control and disturbance compensation techniques". *IEEE Trans. Power Electron*, Vol. 28, Iss. 3, pp. 1358-1365.

[7] I. Omrane, E. Etien, and O. Bachelier (2011), "Reduced-order Luenberger observer for surface-mount permanent-magnets synchronous motors". International conference on Sciences and Techniques of Automatic control & computer engineering STA, Sousse, Tunisia.

[8] A. Barakat, S. Tnani, G. Champenois, E. Mouni (2010), "Analysis of synchronous machine modeling for simulation and industrial applications", *Journal of Simulation Modeling Practice and Theory*, Vol. 18, Iss. 9, pp. 1382-1396.

[9] J. Lee, J. Hong, K. Nam, R. Ortega, L. Praly, A. Astolfi (2010), "Sensorless control of surface-mount permanent-magnet synchronous motors based on a nonlinear observer", *IEEE Trans Power Electron*, Vol. 25, Iss. 2, pp. 290-297.

[10] Ph. Bogaerts, A. VandeWouwer (2003), "Software sensors for bioprocesses", *ISA Trans*, Vol. 42, Iss. 4, pp. 547-558.

[11] R.F. Escobar, C.M. Astorga-Zaragoza, A.C. Tellez-Anguiano, D. Juarez-Romero, J.A. Hernandez, G.V. Guerrero-Ramirez (2011), "Sensor fault detection and isolation via high-gain observers", *ISA Trans*, Vol. 50, Iss. 3, pp. 480-486.

[12] S.R. Vijaya Raghavana, T.K. Radhakrishnan, K. Srinivasan (2011), "Soft sensor based composition estimation and controller design for an ideal reactive distillation column", *ISA Trans*, Vol. 50, Iss. 1, pp. 61-70.

[13] H. Kubota, K. Matsuse, T. Nakano (1993), "DSP-based speed adaptive flux observer of induction motor", *IEEE Trans Ind Appl*, Vol. 29, Iss. 2, pp. 344 -348.

[14] H.R. Karimia, A. Babazadehb (2005), "Modeling and output tracking of transverse flux permanent magnet machines using high gain observer and RBF neural network", *ISA Trans*, Vol. 44, Iss. 4, pp. 445-456.

[15] S. Zheng, X. Tang, B. Song, S. Lu, B. Ye (2013), "Stable adaptive PI control for permanent magnet synchronous motor drive based on improved JITL technique", *ISA Trans*, Vol. 52, Iss. 4, pp. 539-549.

[16] G. Yang, T.H. Chin (1993), "Adaptive-Speed identification scheme for a vector-controlled speed sensorless inverter-induction motor drive", *IEEE Trans Ind Appl*, Vol. 29, Iss. 4, pp. 820-825.

[17] G. Madadi-Kojabadi (2005), "Simulation and experimental studies of model reference adaptive system for sensorless induction motor drive", *Simul Model Pract Theory*, Vol. 13, Iss. 6, pp. 451-464.

[18] T. Orlowska-Kowalska, M. Dybkowski (2010), "Stator-current-based MRAS estimator for a wide range speed-sensorless induction-motor drive", *IEEE Trans Ind Electron*, Vol. 57, Iss. 4, pp. 1296-1308.

[19] D. Xu, S. Zhang, J. Liu (2013), "Very-low speed control of PMSM based on EKF estimation with closed loop optimized parameters", *ISA Trans*, Vol. 52, Iss. 6, pp. 835-843.

[20] M. Hinkkanen, M. Harnefors, J. Luomi (2010), "Reduced-order flux observers with stator-resistance adaptation for speed-sensorless induction motor drives", *IEEE Trans Power Electron*, Vol. 25, Iss. 5, pp. 1173-1183.

[21] I. Omrane (2016), "Design of an easy tunable soft sensor for real-time speed and position estimation of PMSM", *International Journal of Modern Communication Technologies & Research*, Vol. 4, Iss. 2, pp. 10-18.

[22] J. Hu, D. Zhu, Y. Li, J. Gao (1994), "Application of sliding observer to sensorless permanent magnet synchronous motor drive system". 25th Annual EEE Power Electronics Specialists Conference PESC, Taipei, Taiwan, p.p. 532-536.

[23] Y. S. Han, J. S. Choi, Y. S. Kim (2000), "Sensorless PMSM drive with a sliding mode control based adaptive speed and stator resistance estimator". *IEEE Trans. Magn*, Vol. 36, Iss. 5, pp. 3588-3591.

[24] C. Jianbo, H. Yuwen, H. Wenxin, W. Mingjin, Y. Jianfei, and S. Yuxia (2009), "An improved sliding-mode observer for position sensorless vector control drive of PMSM". 6th International Power Electronics and Motion Control Conference IPEMC, Wuhan, China, pp. 1898-1902.

[25] L. Wenqi, H. Yuwen, H. Wenxin, C. Jianbo, D. Xuyang, Y. Jianfei (2008), "Sensorless control of permanent magnet synchronous machine based on a novel sliding mode observer". IEEE Vehicle Power and Propulsion Conference VPPC, Harbin, China, pp. 1-4.

[26] G. Ya, L. Weiguo, Y. Qian (2011), "Study of position sensorless control based on sliding mode observer". International Conference on Electrical Machines and Systems ICEMS, Beijing, China, pp. 1-3.

[27] H. Kim, J. Son, J. Lee (2011), "A high-speed sliding-mode observer for the sensorless speed control of a PMSM". *IEEE Trans. Ind. Electron*, Vol. 58, Iss. 9, pp. 4069-4077.

[28] K. Paponpen, M. Konghirum (2006), "An improved sliding-mode observer for speed sensorless vector control drive of PMSM". International Power Electronics and Motion Control Conference IPEMC, Shanghai, China, pp. 1-5.

[29] C. Wei, C. Yankun, L. Hongfeng, S. Zhanfeng (2012), "Sensorless control of permanent magnet synchronous motor based on sliding mode observer". 7th International Power Electronics and Motion Control Conference IPEMC, Harbin, China, pp. 2582-2586.

[30] J. Liu, T.A. Nondahl, P.B. Schmidt, S. Royak, M. Harbaugh (2011). , "Rotor position estimation for synchronous machines based on equivalent EMF", *IEEE Trans. Ind. Appl*, Vol. 4, Iss. 3, pp. 1310-1318.

[31] I. Omrane (2016), "Design of an easy tunable soft sensor for real-time speed and position estimation of PMSM", *International Journal of Modern Communication Technologies & Research (IJMCTR)*, Vol. 4, Iss. 2, pp. 10-18.

[32] M. Rashed, P. F. A. MacConnell, A. Fraser Stronach, P. Acarnley (2007), "Sensorless indirect-rotor-field-orientation speed control of a permanent-magnet synchronous motor with stator-resistance estimation", *IEEE Trans. Ind. Electron*, Vol. 54, Iss. 3, pp. 1664-1675.