

# Heuristic Optimization Algorithm for Peak – to – Average Power Ratio Reduction in Wireless Orthogonal Frequency Division Multiplexing Systems

Ho-Lung Hung, Chung-Hsien Chen

**Abstract**— Orthogonal frequency division multiplexing (OFDM) which has been adopted in the long-term evolution (LTE) system can improve the system capacity obviously. However, one main disadvantage of the OFDM is the high peak-to-average power ratio (PAPR), which can be reduced by using tone reservation (TR). In tone reservation based OFDM systems; the PAPR reduction performance mainly depends on the selection of the peak reduction tone (PRT) set. Finding the optimal PRT set requires an exhaustive search of all combinations of possible PRT sets, which is a nondeterministic polynomial-time (NP-hard) problem, and this search is infeasible for the number of tones used in practical systems. To address this problem, the proposed methods are based on the technique called the harmony search (HS) algorithm is introduced to determine sub-optimum values of the PRTs with low complexity. Compared with the sub-optimization method, the proposed scheme can generate the optimal peak-canceling signals with fast convergence. Our simulation results demonstrate that the TR based on HS algorithm can achieve the good tradeoff between PAPR reduction performance and computational complexity.

**Index Terms**- Long-term evolution (LTE) system, OFDM, Harmony search algorithm, Peak-to-average power ratio reduction, Tone reservation.

## I. INTRODUCTION

Multicarrier transmission represents a direction that most state-of-the-art wireless communication standards evolve toward, including digital video broadcasting, IEEE 802.11, IEEE 802.16, and 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) [1-3] standards. The key technologies such as orthogonal frequency division multiplexing (OFDM) and multi-input multi-output (MIMO) are adopted in LTE system. OFDM has become a popular method to high-rate wireless communications due to its simple implementation, robustness against frequency-selective fading, and relative simplicity when employing multiple-antenna transmission techniques [2-4]. However, one of the major problems of OFDM-based systems is the high peak-to-average-power ratio (PAPR) of a transmitted signal, which causes a distortion of a signal at the nonlinear high-power amplifier (HPA) of a transmitter. Thus,

the power efficiency of the HPA is seriously limited to avoid nonlinear distortion; otherwise, the high PAPR results in significant performance degradation.

Peak-to-average power ratio is widely used to characterize envelope fluctuations of OFDM signals by relating peak and mean power [5-10]. Many PAPR reduction techniques have been proposed [9], [10], such as amplitude clipping [8], Coding [7], and companding [8], tone injection(TI) [9], tone reservation (TR)[10], active constellation extension (ACE) [11], selected mapping (SLM) [12], and partial transmit sequences (PTS) [13]. Among these techniques, TR techniques have gained great attention due to their flexibility and low complexity. An efficient method to reduce the PAPR of OFDM signals is tone reservation (TR). The TR method reserves a number of subcarriers (tones) and maps appropriate non-data symbols to the reserved tones in order to reduce signal peaks in the time domain [9], [10] [14-20]. The TR technique [9] proposed by Tellado is a distortionless method based on using a small subset of subcarriers, called peak reduction tones (PRTs), to generate a peak-canceling signal for PAPR reduction. These methods have been labeled under the category of tone reservation techniques [14-20], and their main strategy is to use these tones at the transmitter-by aid of some clever signal processing- to build peak-reducing signals akin to clippers that are completely confined in their frequency support to the reserved tones, such that they could be clearly discernible and hence removable at the receiver.

The problem of optimum selection of PRTs' values is a convex optimization problem [18-20] that leads to increased computational complexity and low convergence rates. However, finding the optimal PRT set requires an exhaustive search of all combinations of possible PRT sets, which is a nondeterministic polynomial-time (NP-hard) problem, and this search is infeasible for the number of tones used in practical systems. Hence, an alternative approach is to use a genetic algorithm (GA) to select the PRT values and is developed to solve the NP-hard problem for reducing the PAPR of a transmitted OFDM system. The GA-assisted TR scheme proposed in [18] outperforms the iterative TR-based techniques [16] for reducing the PAPR of a transmitted OFDM signal; the performance of the GA-assisted TR scheme can be improved. However, the GA- based TR method might is easily trapped into local solutions. The cross entropy (CE)-based TR method is [17] is another technique. Although it can offer the best PAPR reductions, but it requires larger population or sampling [14]. Optimal PAPR-reduction by TR can be obtained by solving a quadratically constrained

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quadratic problem (QCQP) [9]. However, several low-complexity methods have been proposed [11], [15] that achieve sufficiently accurate suboptimal solutions. On the other hand, the PAPR reduction performance and the computational complexity of TR scheme depend on the method of PRT. These schemes have a trade-off between PAPR reduction performance and computational complexity. The aim of this paper is to demonstrate some sub-optimum solutions to the OFDM PAPR reduction problem, based on the TR concept and by exploiting the intrinsic advantages of Harmony search (HS) [22-26] algorithm. This paper develops an improved harmony search (IHS) algorithm for solving optimization problems. IHS employs a novel method for generating new solution vectors that enhances accuracy and convergence rate of harmony search (HS) algorithm.

In this paper, we first propose a new suboptimal PRT set selection scheme based on HS algorithm, which can efficiently solve the optimal values of the PRTs that minimize the PAPR of the transmitted OFDM signal and NP-hard problem. Simulation results indicate that the proposed HS-based algorithm performs better than the GA-assisted TR scheme [18], some sub-optimum solutions [19] and conventional TR method. This paper is organized as follows: In Section II, OFDM system and TR scheme are described. Section III introduces a new HS-assisted TR scheme for the nearly optimal PRT set and discusses the computational complexity issue. The simulation results are shown in Section IV, and, finally, the concluding remarks are given in Section V.

## II. OFDM SYSTEM MODEL AND PAPR DEFINITION

### A. PAPR definition

In this section, we briefly review the basics of the OFDM transmitter and we define the PAPR for OFDM system. Moreover, an overview of the TR technique is also exposed. In an OFDM system, an input data symbol vector  $\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T$  in the frequency domain is modulated by  $N$  orthogonal subcarriers to generate a discrete time OFDM signal  $x_n$ . The  $N$  subcarriers are chosen to be orthogonal, that is,  $f_m = m\Delta f$ , where  $\Delta f = \frac{1}{NT}$ . The OFDM signal is generated by summing all the  $N$  modulated subcarriers each of which is separated by  $\frac{1}{NT}$ . Then the complex OFDM signal in the time domain is expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i e^{j2\pi ni/LN}, \quad 0 \leq n \leq N-1 \quad (1)$$

where can also be written in matrix form  $\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T = \mathbf{D}\mathbf{X}$ , where  $\mathbf{D}$  is the DFT matrix with the  $(n,k)$  the entry  $d_{n,k} = (1/\sqrt{N})e^{j2\pi nk/LN}$ , where  $j = \sqrt{-1}$ , and  $n$  stands for a discrete time index. When  $L = 1$ , the above equation reduces to the Nyquist rate sampling case.  $L$  is the oversampling factor, where  $L = 4$ , which is enough to provide an accurate approximation of the

PAPR [4] and  $x_n$  is the  $n$ th signal component in OFDM output symbol. For a data frame, the ratio of the maximum peak power to the average power, which is referred to as the peak-to-average-power ratio of OFDM  $x_n$ , is defined as [9][11]

$$PAPR(x) = 10 \log_{10} \frac{\max_{0 \leq n \leq LN-1} |x_n|^2}{E[|x_n|^2]}, \quad (2)$$

where  $\max |x_n|^2$  is the maximum values of the OFDM signal power, and  $E[\cdot]$  is the mathematical expectation. As PAPR is a random variable so it cannot be a single value for a system. Generally PAPR is characterized by Complementary Cumulative Distribution Function (CCDF) defined as

$$CCDF(PAPR_0) = P_r[PAPR > PAPR_0] \quad (3)$$

where  $PAPR_0$  is any positive value and  $P_r[\cdot]$  is probability function. The PAPR reduction performance of TR scheme mainly depends on the size of PRT set, the maximum number of iterations, and the selection of PRTs [17-19]. In order to reduce the PAPR of OFDM signal using TR scheme, some subcarriers are reserved as PRT set which is used to generate peak cancelling signal. For the TR technique, the transmitter does not send data on a small subset of subcarriers that are optimized for PAPR reduction [9][14]. The objective is to find the time domain signal to be added to the original time domain signal  $\mathbf{x}$  such that the PAPR is reduced. It is also known that finding the optimal PRT set is equivalent to finding the time domain kernel with the minimum secondary peak, where the time domain kernel is obtained by inverse fast Fourier transforming (IFFT) the characteristic sequence of the PRT set. The secondary peak minimization problem is known to be nondeterministic polynomial-time (NP)-hard, which cannot be solved for the practical number of tones. Therefore, one has to resort to suboptimal solutions such as random set optimization [14][16-20]. In this paper, we formulate the optimal Peak Reduction Tones (PRT) set selection problem as a constrained combinatorial optimization, and propose the application of the HS algorithm [25-26] to solve the problem. In an OFDM system with  $N$  subcarriers, the TR scheme uses some subcarriers within the  $N$  subcarriers as a PRT set that is reserved for generating PAPR reduction signals. In TR subcarriers, called Peak Reduction Tones (PRT's), are set aside for PAPR reduction as shown in the transceiver block diagram in Figure 1. In practice, TR algorithms seek the best choice of  $C$  to decrease the peak power of the modified waveform. In the TR-based OFDM scheme, peak reduction tones are reserved to generate PAPR reduction signals. These reserved tones do not carry any data information, and they are only used for reducing PAPR. Specifically, the peak-cancelling signal  $\mathbf{C} = [C_0, C_1, \dots, C_{N-1}]^T$  generated by reserved PRT is added to the original time domain signal  $\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T$  to reduce its PAPR. The PAPR reduced signal can be expressed as [14-18]

$$\mathbf{x} + \mathbf{c} = \mathbf{Q}(\mathbf{X} + \mathbf{C}) \quad (4)$$

where  $\mathbf{Q}$  is the IFFT matrix,  $\mathbf{X}$  is the transmit data before the IFFT, and  $\mathbf{C}$  are the PRT's.

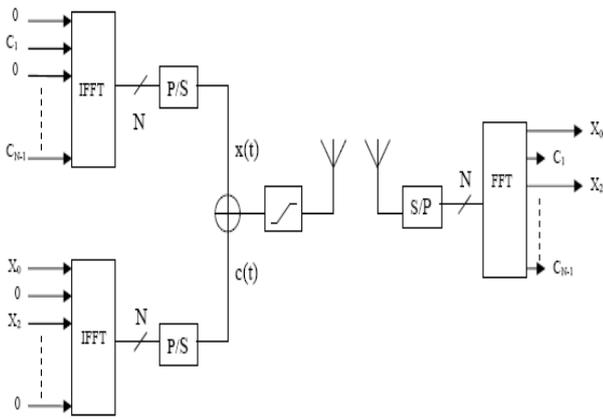


Figure 1 Block diagram of a Tone Reservation (TR) OFDM transceiver.

where  $\mathbf{X}$  and  $\mathbf{C}$  denote the signals transmitted on the data tones and reserved tones, respectively. The TR scheme uses two indices  $R$  and  $M$  to assign the subcarriers used as transmitting the data symbols  $\mathbf{X}$  or the PAPR reduction symbols  $\mathbf{C}$ . The transmitted symbols consist of the data symbols  $\mathbf{X}$  and the PAPR reduction vector  $\mathbf{C}$ . The  $\mathbf{C}$  is the time domain due to  $\mathbf{C}$ . The TR technique restricts the data block  $\mathbf{X}$  and frequency domain vector  $\mathbf{C} = [C_0, C_1, \dots, C_{N-1}]^T$  is avoid signal distortion,  $\mathbf{X}$  and  $\mathbf{C}$  are orthogonal with each other, the transmitted symbol  $S_n$  can be written as

$$\mathbf{X}_n + \mathbf{C}_n = \begin{cases} \mathbf{X}_n, & n \in M, \\ \mathbf{C}_n, & n \in R, \end{cases} \quad (5)$$

where  $M = \{m_0, m_1, \dots, m_{N-W-1}\}$  represents the index set of the data-bearing subcarriers and  $R = \{r_0, r_1, \dots, r_{W-1}\}$  is the index set of the reserved tones. In addition,  $R$  is also referred to as a PRT set and  $W < N$  is the number of PRT. Notably, PRTs are not used to transmit information, but rather to generate PAPR reduction signals. In [9], [11], the PAPR of the OFDM signal sequence is defined  $S$  as

$$PAPR(S) = \frac{\max_{0 \leq n \leq N-1} |x_n + c_n|^2}{E[|x_n|^2]} \quad (6)$$

Basically, we can adjust the value of  $C_n$  to reduce the peak value of  $x_n$  without disturbing the actual data contained in  $X$ . Thus,  $C_n$  must be investigated to minimize the maximum norm of the time domain signal  $x_n$ , The optimization problem is to find optimum  $c$  according to

$$c^{(opt)} = \arg \min_C \max_{0 \leq n < N} |x_n + c_n|^2 \quad (7)$$

Since subcarriers in OFDM systems are orthogonal, and the vectors added are restricted by (7), signals in spare subcarriers can be simply discarded at the receiver. Accordingly, in an exhaustive search approach, the computational complexity increases exponentially with the

number of PRTs. Then a minima PAPR optimization problem is formulated as

$$\min_{C \in \Pi} E, \quad (8)$$

$$\text{s.t. } |x_n + c_n|^2 \leq E, \text{ for all } n = 0, 1, \dots, N-1,$$

where  $E$  represents the peak power of the peak-reduced signal, and  $\Pi$  is the signal space of all possible peak-compensation signals generated from the set of  $R$ . In order to find the optimal values of the PRT, the TR optimization problem can be formulated as a QCQP problem. To reduce the complexity of the QCQP, Tellado in [9] proposed a simple a simple gradient algorithm to iteratively approach  $c$  and updates the vector as follows:

$$\mathbf{c}^{(i+1)} = \mathbf{c}^{(i)} - \beta_i \mathbf{w} \left[ \left( (k - k_i) \right)_n \right] \quad (9)$$

where  $\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T$  is called the time domain kernel,  $\beta_i$  is a scaling factor relying on the maximum peak found at the  $i$ th iteration, and  $\mathbf{w} \left[ \left( (k - k_i) \right)_n \right]$  denotes a circular shift of  $\mathbf{w}$  to the right by a value  $k_i$ , where  $k_i$  is calculated by

$$k_i = \underset{k}{\operatorname{argmax}} |x_k + c_k^{(i)}| \quad (10)$$

For ease of presentation, the time domain kernel is expressed as

$$\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T = D\mathbf{W} \quad (11)$$

where  $\mathbf{W} = [W_0, W_1, \dots, W_{N-1}]^T$ ,  $W_n \in \{0, 1\}$  is called the frequency domain kernel whose elements are defined by

$$W_n = \begin{cases} 0, & n \in M \\ 1, & n \in R \end{cases} \quad (12)$$

Then, the optimal frequency domain kernel  $\mathbf{W}$  corresponds to the characteristic sequence of the PRT set  $R$  and the maximum peak  $\mathbf{W}$  of is always because is a  $\{0, 1\}$  sequence. The PAPR reduction performance depends on the time domain kernel  $\mathbf{W}$  and the best performance can be achieved when the time domain kernel  $\mathbf{w}$  is a discrete impulse because the maximum peak can be cancelled without affecting other signal samples at each iteration. If the maximum number of iterations is reached or the desired peak power is obtained, iteration stops. For simplicity, we assume that only one maximum peak of the OFDM signal is reduced per iteration in (10). After  $G$  iterations of this algorithm, the peak-reduced OFDM signal is obtained:

$$S = \mathbf{x} + \mathbf{c}^{(G)} = \mathbf{x} - \sum_{g=1}^G \beta_g \mathbf{w} \left[ \left( (k - k_i) \right)_n \right] \quad (13)$$

The PAPR reduction performance of the TR scheme depends on the selection of the PRT set  $R$ . But, this problem is known as NP-hard, because the time domain kernel  $\mathbf{w}$  must be optimized over all possible discrete sets  $R$  [14-16]. Thus, it is cannot be solved for practical values of  $N$ . To find the optimal PRT set, in mathematical form, we require solving the following combinatorial optimization problem:

$$R^* = \arg \min_R \left\| [w_1, w_2, \dots, w_{N-1}]^T \right\|_{\infty} \quad (14)$$

where  $\|\cdot\|_j$  denotes the  $j$ -norm and  $\infty$ -norm refers to the maximum values. Here,  $R^*$  denotes the global optimum of the objective function, which requires an exhaustive search of all combination of possible PRT set  $R$ . All practical attempts at implementing TR solve some related approximation to this problem, trading optimal performance for manageable computational complexity.

**B. Harmony search Algorithm Based PRT set Search for reduced PAPR**

The Harmony search (HS) algorithm is a new metaheuristic population search algorithm proposed by Geem *et al.* [22-23]. HS was derived from the natural phenomena of musicians' behavior when they collectively play their musical instruments) to come up with a pleasing harmony (global optimal solution). This state is determined by an aesthetic standard (fitness function). It has received considerable attention regarding its potential as an optimization technique and has been successfully applied in various areas [25-26]. Furthermore, in comparison with traditional optimization methods, HS has several advantages. It imposes fewer mathematical requirements and produces a new solution after considering all the existing solutions [23]. When applied to optimization problems, the musicians typically represent the decision variables of the cost function, and HS acts as a meta-heuristic algorithm which attempts to find a solution vector that optimizes this function. In such a search process, each decision variable (musician) generates a value (note) for finding a global optimum (best harmony). The HS algorithm, therefore, has a novel stochastic derivative (for discrete variable) based on musician's experience, rather than gradient (for continuous variable) in differential calculus.

Harmony memory contains harmonies played by the musicians, or it can be viewed as the storage place for solution vectors. Original HS uses five parameters, including three core parameters such as the size of harmony memory (**HMS**), the harmony memory considering rate (**HMCR**), and the maximum number of iterations  $K$ , and two optional ones such as the pitch adjustment rate (**PAR**) and the adjusting bandwidth (later developed into fret width) (**BW**). The number of musicians  $N$  is defined by the problem itself and is equal to the number of variables in the optimization function. The detailed usage of these parameters, along with the iteration steps, is illustrated in the following sections.

A detailed description of the HS algorithm used for searching the nearly optimal PRT set positions is described in what follows. An initial population of musician is randomly generated. Each tone sequences is a vector of length  $N$ , and each element of the vector is a binary zero or one depending on the existence of a PRT set at that position (one denotes existence and zero denotes non-existence). The number of the PRT in each binary vector is  $M < N$ . Denote the  $V$  tones as  $\{v_1, v_2, \dots, v_s\}$ . Then each  $\mathbf{V}_u$  is a binary vector of length.

For each tones  $\mathbf{V}_u$ , the PRT set  $R_u$  is the collection of the locations whose elements are one. Then the frequency domain kernel corresponding to the PRT set is obtained by (12), and the time domain kernel  $W_u = [w_0^u, w_1^u, \dots, w_{N-1}^u]$  is obtained by (9). The merit (secondary peak) of the sequence is defined as [14-17]

$$m(\mathbf{V}_u) = \left\| \left[ w_1^u, w_2^u, \dots, w_{N-1}^u \right]^T \right\|_{\infty} \quad (15)$$

The  $T$  sequences (called elite sequences) with the lowest merits are maintained for the next population generation. The best merit of the sequences is defined as

$$\hat{m} = \min_{1 \leq u \leq N} m(\mathbf{V}_u) \quad (16)$$

**III. MINIMIZE PAPR USING HARMONY SEARCH ALGORITHM-BASED PRT SET**

In this section, we will briefly introduce the HS and use it to search for a nearly optimal PRT set. A harmony in music is analogous to the optimization solution vector, and the musician's improvisations are analogous to the local and global search schemes in optimization techniques. The HS algorithm uses a stochastic random search, instead of a gradient search. This algorithm uses harmony memory considering rate and pitch adjustment rate for finding the solution vector in the search space. In the HS algorithm uses the concept, how aesthetic estimation helps to find the perfect state of harmony, to determine the optimum value of the objective function. The HS algorithm is simple in concept, few in parameters and easy in implementation. It has been successfully applied to various optimization problems.

The key parameters which have a profound effect on the HS performance are harmony memory considering rate (HMCR), pitch adjusting rate (PAR) and bandwidth of generation (bw). These parameters can be potentially useful in adjusting convergence rate of the algorithm to optimal solution. A harmony is selected from the HM with the probability of 1-HMCR. It is introduced to get away from local optima when all parts of the global solution do not exist in the HM. The value of PAR will determine the probability of generating a value near-by one value chosen from the HM. This parameter is used to improve the quality of the HM harmonies and bw provides a balance between local and global search. HMCR value varies between 0 and 1. According to the above algorithm concept, the HS met heuristic algorithm consists of the following five steps [[23-26]:

**Step 1. Initialization of the optimization problem and algorithm parameters:** this process is only executed at the first iteration. In the first step, a possible value range for each design variable of the optimum design problem is specified. A pool is constructed by collecting these values together, from which the algorithm selects values for the design variables. A possible value range for each design variable of the optimum design problem is specified. Furthermore, the number of solution vectors in harmony search memory that is the size of the harmony memory matrix, PAR and the maximum number of searches are also selected in this step.

$$\text{Minimize } f(\hat{x})$$

$$\text{subject to } x_i \in X_i, \quad i = 1, 2, \dots, M \quad (17)$$

where  $f(\cdot)$  is a scalar objective function to be optimized,  $\hat{x}$  is a solution vector composed of decision variables  $x_i$ ,  $X_i$  is the set of possible range of values for each decision variable  $x_i$ , that is,  $x_i^L \leq x_i \leq x_i^U$ , where  $x_i^L$  and  $x_i^U$  are the lower and upper bounds for each decision variable, respectively,

and  $M$  is the number of decision variables. Randomly generate an initially feasible population of size, and find the PRT set for each sequence. Calculate the frequency domain kernel using (12) and the time domain kernel using (11) for each sequence. In addition, the control parameters of HS are also specified in this step. These parameters are the HM size (HMS) i.e., the number of solution vectors (population members) in the HM (in each generation); the HMCR; the PAR; and the number of improvisations (NI) or stopping criterion. The harmony memory (HM) is a memory location where all the solution vectors (sets of decision variables) are stored. This HM is similar to the genetic pool in the GA. Here, HMCR and PAR are parameters that are used to improve the solution vector. Both are defined in Step 3.

**Step 2.** The harmony memory matrix is initialized. Each row of the harmony memory matrix contains the values of design variables which are randomly selected feasible solutions from the design pool for that particular design variable. HMS is similar to the total number of individuals in the population matrix of the genetic algorithm. In this step, the ‘‘harmony memory’’ matrix shown in Eq. (16) is filled with as many randomly generated solution vectors as the size of the HM (i.e., HMS) and sorted by the values of the objective function,  $f(x)$

$$HM = \begin{bmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ x_{1,HMS-1} & x_{2,HMS-1} & \cdots & x_{n,HMS-1} \\ x_{1,HMS} & x_{2,HMS} & \cdots & x_{n,HMS} \end{bmatrix} \quad (18)$$

where  $x_{i,j}$  is the value of the  $i$ th design variable in the  $j$ th randomly selected feasible solution. These candidate designs are sorted such that the objective function value corresponding to the first solution vector is the minimum. In other words, the feasible solutions in the harmony memory matrix are sorted in descending order according to their objective function value. It is worthwhile mentioning that not only the feasible designs are inserted into harmony memory matrix, but those designs having a small infeasibility are also included in this matrix with a penalty on their objective function.

**Step 3.** New harmony memory matrix is improvised.

In generating a new harmony matrix the new value of the  $i$ th design variable can be chosen from any discrete value within the range of  $i$ th column of the harmony memory matrix with the probability of HMCR which varies between 0 and 1. In other words, the new value of  $x_i$  can be one of the discrete

values of the vector  $\{x_{i,1}, x_{i,2}, \dots, x_{i,HMS}\}^T$  with the probability of HMCR. The same is applied to all other design variables. In the random selection, the new value of the  $i$ th design variable can also be chosen randomly from the entire pool with the probability of  $1 - HMCR$ . That is,

$$x_i^{new} = \begin{cases} \{x_{i,1}, x_{i,2}, \dots, x_{i,HMS}\}^T & \text{with probability HMCR} \\ \{x_1, x_2, \dots, x_{ns}\} & \text{with probability (1-HMCR)} \end{cases} \quad (19)$$

where  $ns$  is the total number of values for the design variables in the pool. The PAR decides whether the decision variables are to be adjusted to a neighboring value. The number of improvisations (NI) corresponds to the number of iterations. If the new value of the design variable is selected among those of harmony memory matrix, this value is then checked to see whether it should be pitch adjusted. This operation uses the pitch adjustment parameter PAR that sets the rate of adjustment for the pitch chosen from the harmony memory matrix as follows [22-25]:

$$\text{Pitch adjusting decision for } x_i^{new} \leftarrow \begin{cases} \text{yes} & \text{with probability of PAR} \\ \text{no} & \text{with probability of (1-PAR)} \end{cases} \quad (20)$$

Supposing that the new pitch-adjustment decision for  $x_i^{new}$  came out to be yes from the test and if the value selected for  $x_i^{new}$  from the harmony memory is the  $k$ th element in the general discrete set, then the neighboring value  $k+1$  or  $k-1$  is taken for new  $x_i^{new}$ . This operation prevents stagnation and improves the harmony memory for diversity with a greater chance of reaching the global optimum. In Step 3, HM consideration, pitch adjustment or random selection is applied to each variable of the new harmony vector in turn.

**Step 4.** Update harmony memory: After selecting the new values for each design variable, the objective function value is calculated for the new harmony vector. If the new harmony vector is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order by the objective function value.

**Step 5.** Check stopping criterion. Steps 3 and 4 are repeated until the termination criterion, which is the pre-selected maximum number of cycles, is reached. This number is selected large enough such that within this number of design cycles no further improvement is observed in the objective function.

#### IV. SIMULATION RESULTS

To evaluate and compare the performance of HS based nearly optimal PRT set positions searching and the IHS-TR algorithm for OFDM PAPR reduction, extensive simulations have been conducted. The simulation results are performed using an IEEE802.11a/g wireless local area network (WLAN) standard. In 802.11a/g WLAN standard, the IFFT size ( $N$ ) is 64. Simulation experiments are conducted in this section to verify the PAPR performance of the proposed HS-based TR scheme. We assume OFDM systems with subcarriers  $N = 64, 128, 256$  and 1024, in which the transmit signal is oversampled by a factor of 4 and  $10^4$  OFDM blocks are generated randomly to obtain the complementary cumulative distribution function of the PAPR. The PAPR reduction performance is evaluated in terms of its complementary cumulative distribution function,  $CCDF = (PAPR > PAPR_0)$  which is the probability that

the PAPR of a symbol exceeds the threshold level  $PAPR_0$ . The parameter setting for HS based algorithms is  $HMS = 30$ ,  $HMCR = 0.9$ ,  $PAR_{max} = 0.6$ ,  $PAR_{min} = 0.2$ ,  $bw_{max} = 1$ ,  $bw_{min} = 0.0001$ ,  $t_{max} = 5000$  and the number of the elite harmonies used in IHS is set to 5. Next, we elaborate the step-by-step procedure of the proposed HS algorithm to search for the optimal that minimizes the PAPR as follows.

STEP.1. Select the values of harmony parameters. The harmony memory size HMS, the harmony memory considering rate HMCR and the pitch-adjustment rate PAR are selected as 30, 0.9 and 0.6 respectively in this study. These values are decided after carrying out several trials on the design examples.

STEP.2. Generate a harmony memory matrix. Select randomly sequence number of a steel section from the discrete list for each group in the frame.

STEP.3. Carry out the analysis of the steel frame with these sections and check whether the design limitations are satisfied or not. If the design vector violates the design constraints severely discard the vector and repeat the selection of a new design vector. If it is slightly infeasible consider it for the harmony memory matrix.

STEP.4. Check whether the new design vector selected should be pitch-adjusted as explained in step 3 of the harmony search method. Calculate the objective function value for the newly selected design vector. If this value is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The slightly infeasible designs are inserted into the harmony memory matrix with a penalty on their objective function values. The harmony memory matrix is then sorted in descending order by the objective function values.

STEP.5. Repeat steps 2 and 5 until the termination criterion which is the pre-selected maximum number of cycles is reached. This number is selected large enough so that within this number of design cycles no further improvement is observed in the objective function.

We simulated an OFDM system with subcarriers, where  $W = 32$  and  $W = 64$  tones are, respectively, reserved for PRT set, and the remaining tones are used to transmit data symbols. For comparison, the random set optimization [14], [17] is also tested; the random optimal PRC set is obtained by generating  $10^7$  random sets and choosing the best set. Let us now proceed to the application of HS based tone reservation method in OFDM PAPR reduction. In order to generate the complementary cumulative distribution function (CCDF)  $CCDF = (PAPR > PAPR_0)$  of the PAPR,  $10^{-4}$  OFDM blocks are generated randomly, where the transmitted signal is over-sampled by a factor of  $L = 4$ .

Fig. 2 plots the average PAPR reduction performance of the HS, IHS-TR, GA-TR [18] and Newton-TR [19] with PRT set for the same iteration numbers. From the figure, it can be seen that both the Newton-TR and GA-TR method and our optimization method can significantly reduce the PAPR while simultaneously achieving a comparable sharp drop in their number of iterations. In the following, we provide the simulation results showing that the PAPR of the OFDM signal: 1) it is beneficial for maximum number of iterations

was increased, and the CCDF of the average PAPR reduction performance has been improved and 2) the plots also show that GA and Newton require higher complexity to obtain the same average PAPR reduction performance as the proposed IHS. 3) give a fixed number of iterations, the figures illustrate that IHS has better average PAPR reduction performance than GA- and Newton-TR employed. Just as expected, the PAPR performance of our proposed IHS-based TR scheme with 8 iteration, is not only almost the same as that of the GA-based TR scheme with  $G_{en} = 50$  iteration when average PAPR=7.2dB, but also having much lower computational complexity. However, the computational load of the IHS is only 64 ( $(W \times Iter = 8 \times 8)$ ), which mean that the IHS method can offer good PAPR reduction while maintaining low complexity. In general, in order to obtain optimal PAPR search for the number of iterations must be accomplished. It is can be found that the performance of the IHS-PRT algorithm is better than that of the GA-PRT one in approximately 10 iterations. Most importantly, as the increase of iterations, the secondary peaks of the IHS-PRT algorithm are improved.

One more observation from the simulation is: the IHS-TR HMCR (Harmony Memory Considering Rate 0.7) algorithm converges to 6.15 dB PAPR in  $G_{en} = 50$  iterations, however, which is approximately 0.1 dB larger than IHS-TR HMCR (0.9) algorithm in the same iterations. Note that the PAPR performance of the IHS harmony memory considering rate (0.5) algorithms and the Newton method [19] is close when the iteration number equals 50 with 6.8 dB and 6.9 dB. This shows that adding flexibility to the choice of the number of iterations and harmony memory considering rate can improve the convergence speed of the algorithm. In addition, at the same PAPR reduction, the IHS-assisted TR method required less complexity compared to GA-assisted TR method and Newton- TR method. So we should select a bigger step size to gain better PAPR performance for the IHS-TR algorithm. Finally, this paper shows the trade-off between number of iterations and HMCR for the PAPR reduction.

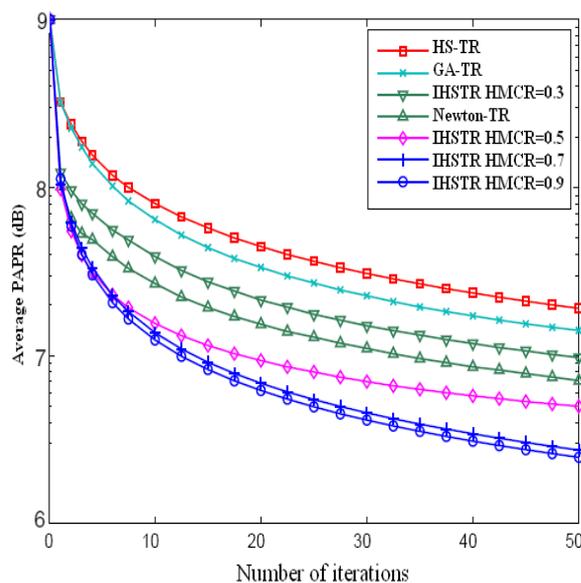


Fig.2 Relationship of PAPR reduction with number of iterations and HMCR for different techniques when N=64.

Fig. 3 shows the PAPR complementary cumulative distribution functions (CCDFs) of OFDM with  $N=64$  at 16-QAM modulation, the number of the reserved tones  $W=8$  employing random partition and numbers of iterations are used for PAPR reduction. The PAPR of the original OFDM signal without the PAPR reduction scheme is about 11.3 dB at  $CCDF = (PAPR > PAPR_0) = 10^{-4}$ . The PAPR reduction performance of the proposed HS-TR scheme is slightly worse than that of the GA-TR[18] scheme for QPSK. The TR scheme with the proposed HS reduces the PAPR by 10 dB at  $10^{-4}$  while the TR scheme with the adjacent IHS parameter reduces the PAPR only by 6.3 dB at  $CCDF = (PAPR > PAPR_0) = 10^{-4}$ . In addition, when  $CCDF = (PAPR > PAPR_0) = 10^{-4}$ ,  $PAPR_0$  the of the original OFDM, HS, GA-, Newton- and IHS- TR are 11.3 dB, 10.8 dB, 8.1 dB, 7.7 dB and 6.30 dB with  $G_{en} = 50$  iterations, respectively. Our scheme has 1.8 dB improvements over GA-TR [18] to reduce PAPR. The result is in line with our expectation.

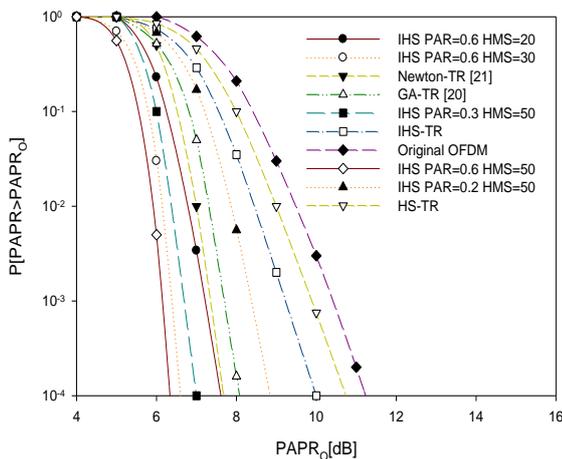


Fig. 3 Comparison of the PAPR CCDF of the IHS method with different step size PAR and HMS for  $N=64$ ,  $W=8$ .

Fig. 4 compares the PAPR reduction performance of the proposed IHS-TR algorithm with the PAR and HMS, GA-TR method in [18], and Newton-TR algorithm [19] for the same conditions of initial population of candidate solutions and number of iterations. As shown in Fig. 4, as the maximum number of PAR and HMS is increased, then, the CCDF of the PAPR has a better performance. Fig.4 compares the PAPR reduction performance of the proposed IHS algorithm based PRC sets, GA-, and Newton-TR algorithm based PRT sets for the same iterations. The same maximum iteration number is set to 50 for GA-TR, Newton-TR, and IHS-TR. When  $CCDF = (PAPR > PAPR_0) = 10^{-4}$ , the PAPR of the original OFDM is 11.4 dB. Using the HS algorithm, GA-TR algorithm, Newton-TR algorithm and IHS-TR algorithm with 128 subcarrier, the PAPRs are approximately reduced to 10.0 dB, 9.0 dB, 8.9 dB, and 6.9 dB, respectively. So the proposed IHS-TR set search algorithm is more efficient than the GA- and Newton-TR algorithm for search the PRT. The fact that the PAPR reduction performance of GA and Newton

algorithm are almost the same will be demonstrated in Fig.4. Compared to Original OFDM, our scheme has about 4.3 dB improvement in PAPR reduction.

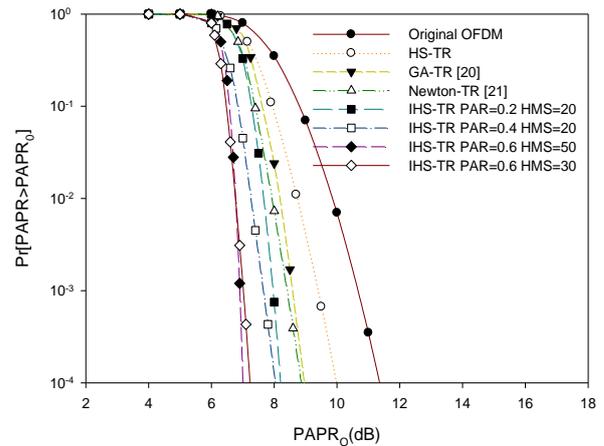


Fig. 4 Comparison of PAPR reduction between GA-TR and Newton-TR for  $N=128$  and  $W=16$ .

Figs. 5 and 6 shows a comparison of the CCDF of PAPR for the conventional OFDM, the GA-TR, the Newton-TR, and the proposed IHS-TR in an OFDM system with  $N = 256$  and 1024 subcarriers for different step size PAP and HMS, respectively. When  $PAR=0.2$ , and  $0.6$ , the PAPR are 8 dB, and 7.5 dB. So we should select a bigger step size PAR to gain better PAPR performance for the IHS-TR algorithm. Next, a fair comparison of the performance-complexity tradeoffs for the different HMS and PAR searching strategies with  $N$  and  $W$ , respectively, is provided in Figs. 5 and 6, where the average PAPR reduction is the function of the number of sample. It can be seen that 1) it is beneficial to select more samples so that the PAPR reduction performance can be improved; and 2) the application of the IHS method to solve the optimum PRT set problem yields an enhanced tradeoff in the low average PAPR range about 7 and 8.92 dB for  $W=16$  and  $W=32$ , respectively. It demonstrates that the IHS-TR algorithm is sensitive to HMS and PAR. Different HMS and PAR ratios can results in different PAPR reduction performance. Note that if the CCDF curve or the allowable PAPR distortion probability is changed by the different number of subcarriers and/or HPA, the PAP and HMS can also be predetermined by employing the above decision criterion.

In the population-based search method such as IHS, GA [18] and Newton-TR [19] method, the number of sample is fixed to find the suboptimal solution with the complexity. In this case, the complexity for GA and Newton-TR could be expressed in terms of the number of samples, where each sample is calculated using the  $N$ -point IFFT. The PAPR obtained by the IHS, GA and Newton-TR with the search computational complexity is defined: the number of samples for HIS, GA and Newton-TR are  $(P * G_{en} * N * L)$ ,  $R * N * L \log(NL)$  and  $(P * G_{en} * N * L \log(NL))$ ; where  $P$  is the size of the population,  $N$  is number of the carrier,  $L$  is the oversampling factor,  $G_{en}$  is the number of generations and  $R$  is the number of reserved tones.

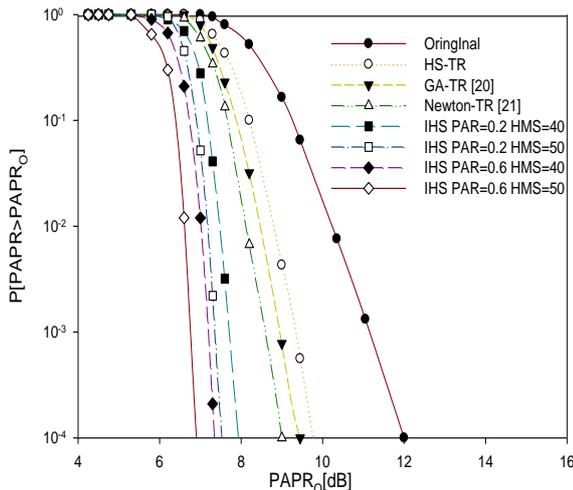


Fig. 5 Comparison of the PAPR CCDF of the IHS method with different step size PAR and HMS for  $N=256$  and  $W=16$ .

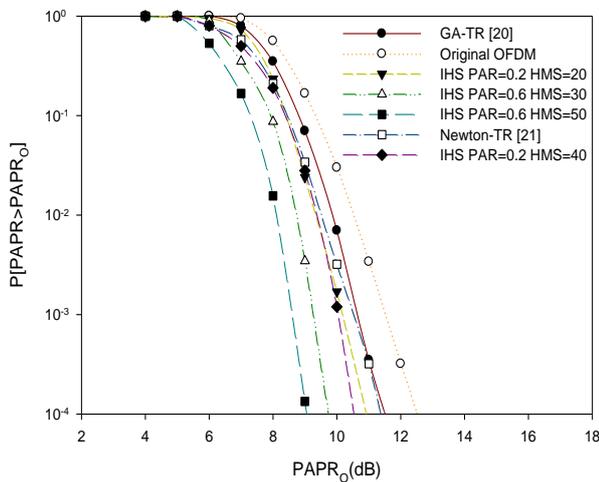


Fig. 6 Comparison of the PAPR CCDF of the IHS method with different step size PAR and HMS for  $N=1024$  and  $W=32$ .

## V. CONCLUSIONS AND DISCUSSIONS

In this paper, we have proposed an improved TR scheme, which is based on the harmony search algorithm, to reduce the PAPR of OFDM signals. The PAPR reduction mainly depends on the selection of peak reduction tone (PRT) set. Compared to the Newton -PRT algorithm and GA-PRT algorithm, the proposed HS-PRT algorithm has lower computational complexity and achieves a good approximation to the secondary peak of the Newton-PRT algorithm. Most importantly, the PAPR performance improves with an increase in the number of PRTs for all the PRT sets. To reduce the computational complexity while still obtaining the desirable PAPR reduction, we introduce the IHS method, an effective algorithm that solves various combinatorial optimization problems, to search the optimal PRTs. The simulations demonstrate that the proposed IHS-based TR scheme not only provide better PAPR

performance but also enjoys complexity advantages compared with the conventional GA-assisted TR method and Newton-TR methods.

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