The modes adaptation function in a rectangular waveguide using an integrated taper metamaterial at microwave bandwidth

A. Qarchi, H. Belkebir, A. Mir, A. Bouzid

Abstract— **The tapers are unavoidable components in the field of telecommunication especially in integrated optics and optoelectronics. They provide a smooth signal transition between different sizes waveguides without affecting the guided modes. In this work, we propose to perform the adaptation function of modes using metamaterials tapers exploiting the compression of Cartesians coordinates technic. The technique that has been used for the first time for highlighting the cloaking phenomenon is based upon the invariance of Maxwell equations. The performances of a conventional taper are faced to metamaterials ones designed from a linear, exponential and parabolic transformation of the Cartesians coordinates in terms of transmission rate.**

Index Terms— **Metamaterials, Permeability, Permittivity, Taper, Waveguide.**

I. INTRODUCTION

Since their appearance in the late 90s, metamaterials continue to develop in several research areas: negative refraction based on the overlapping of metallic wire strips structure [1] that models a negative permittivity within a resonant structure [2] that models a negative permeability. This is the concept on which is based the Super-lenses [3-6] which would free of the resolution limitations imposed by the Rayleigh criterion. The high impedance surfaces [7-9] which become a major alternative to replace an antenna's conventional ground plane in order to enhance its performances. The Cloaking concept [10-15] based on the Cartesians coordinates compression using the invariance of the Maxwell equations [16-17] which aims to control the path of light, and to force the electromagnetic wave to get around an object to make invisible.

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In telecommunication's field, the compression of Cartesians coordinates method allows the design of various optoelectronic components, like the radiation concentrators [18], the beam splitters and the amplifiers [19-21], the polarizers [22] and the high directivity antennas [23-26].

In this work, we intend to show numerically the possibility of designing a metamaterial taper which differs from conventional ones using the compression of Cartesians coordinates technic. Our component aims to guide an electromagnetic wave modes Polarized TE between two different width waveguides. We will investigate the performances of these types of tapers by comparing their transmission against to that of conventional ones.

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II. MODELING

From The compression of Cartesians coordinates, we can extract the new permeability and permittivity tensors relating to this transformation. Indeed, the Maxwell equations expressed in Cartesians coordinates (x, y, z) as follows:

$$\nabla X\boldsymbol{E} = -\mu_0\mu\frac{\partial \boldsymbol{H}}{\partial t} \qquad \nabla X\boldsymbol{H} = \varepsilon_0\varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

remain invariant if one proceeds to a change of reference to a coordinates (x ', y', z ') as follows:

$$\nabla' X\boldsymbol{E}' = -\mu_0\mu'\frac{\partial \boldsymbol{H}'}{\partial t} \qquad \nabla' X\boldsymbol{H}' = \varepsilon_0\varepsilon'\frac{\partial \boldsymbol{E}'}{\partial t}$$

Such as :

$$\boldsymbol{E}' = (A^T)^{-1}\boldsymbol{E} \qquad \boldsymbol{H}' = (A^T)^{-1}\boldsymbol{H}$$

$$\mu' = \frac{1}{det\,A}A\mu A^T \qquad \varepsilon' = \frac{1}{det\,A}A\varepsilon A^T$$

$$A_{ij} = \frac{\partial x'_i}{\partial x_j} \qquad A_{ij}^{-1} = \frac{\partial x_i}{\partial x'_j}$$

Three types of transformations of Cartesians coordinates are proposed:

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- A linear one (Figure 1). The new coordinates satisfy the following equations:

$$x' = x$$
$$y' = \frac{y}{a}\left(\frac{b-a}{l}x + a\right)$$
$$z' = z$$

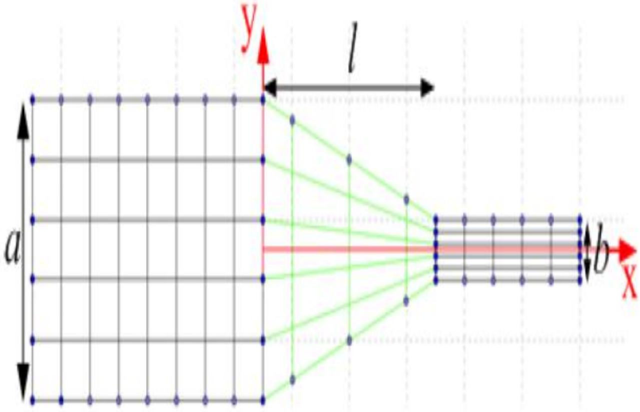

Figure 1: Illustration of liner transformation of Cartesians coordinates

- An exponential transformation (Figure 2). The new coordinates are expressed in terms of the olds as follows:

$$x' = x$$
$$y' = y\left(\frac{b}{a}\right)^{\frac{x}{l}}$$
$$z' = z$$

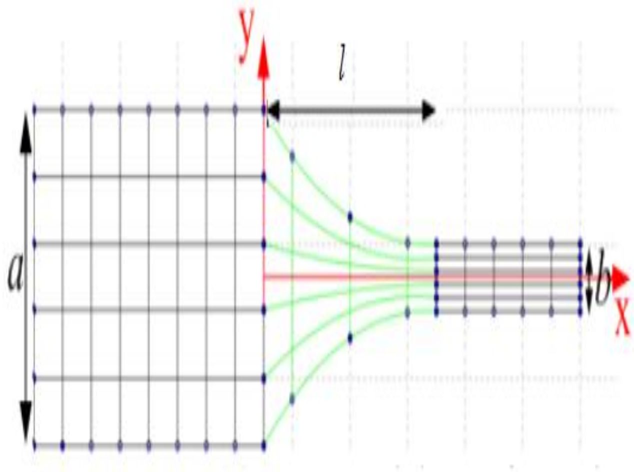

Figure 2: Illustration of exponential transformation of Cartesians coordinates

- A parabolic transformation (Figure 3). The new coordinates are connected via the olds as:

$$x' = x$$
$$y' = \frac{y}{a}\left(\frac{b-a}{l^2}x^2 + a\right)$$
$$z' = z$$

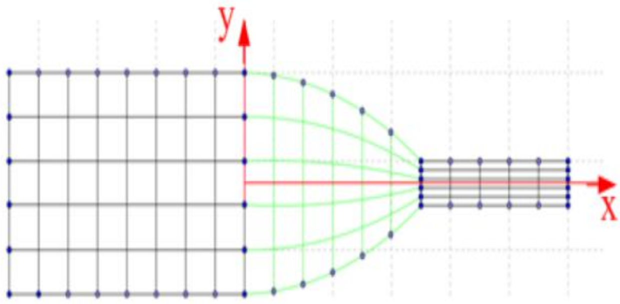

Figure 3: Illustration of parabolic transformation of Cartesians coordinates

The table below shows the values of the permeability μ' and permittivity ε' tensors for the three types of transformations [27]:

$$\varepsilon' = \bar{\bar{\theta}}\varepsilon_0 \qquad \mu' = \bar{\bar{\theta}}\mu_0$$

$$\bar{\bar{\theta}} = \begin{pmatrix} \theta_{xx}(x') & \theta_{xy}(x',y') & 0 \\ \theta_{xy}(x',y') & \theta_{yy}(x',y') & 0 \\ 0 & 0 & \theta_{zz}(x') \end{pmatrix}$$

III. Simulation and Discussion

The input waveguide is a rectangular one width of 10 cm which is connected by a taper length "l" to a rectangular waveguide width of 3 cm. By commercial software that uses the finite elements calculation method, we try to check the phenomenon of adaptation of modes TE_1, TE_2 and TE_3 between the two waveguides. The working frequency is 16 GHz to ensure the excitation of the third mode of the output waveguide since its cutoff frequency is around 15 GHz. Is shown below the electric field distribution of the different modes and the different types of transformation (figures 4, 5, 6).

By integrating the medium obtained by the Cartesians coordinates transformation between the two waveguides, and in three cases of transformation: linear, exponential and parabolic ones, the electromagnetic wave is perfectly guided with great fluidity, from the input waveguide to the output one without suffering of any reflection inside the integrated environment or in the waveguide-tapers interfaces, except some phase distortions to the second interface. So from the quantitative point of view, the metamaterials taper can be one of means to ensure the connection between the different widths waveguides since the transition of modes is correctly conserved.

From a qualitative point of view, to verify the performances of this kind of tapers, a parametric study is done where we vary the taper length between 2.5 cm and 5 cm in order to check the better signal coupling rate for the shorter taper length between the input waveguide and the output.

	$\theta_{xx}(x')$	$\theta_{xy}(x',y')$	$\theta_{yy}(x',y')$
Linear transformation	$\frac{al}{a(l-x')+bx}$	$\theta_{xx}^2 \frac{b-a}{al} y'$	$\frac{1}{\theta_{xx}} + \frac{\theta_{xx}^2}{\theta_{xy}}$
Exponential transformation	$\left(\frac{b}{a}\right)^{\frac{-x'}{l}}$	$\frac{\theta_{xx}\, y' \log\left(\frac{b}{a}\right)}{l}$	$\frac{1}{\theta_{xx}} + \frac{\theta_{xx}^2}{\theta_{xy}}$
Parabolic transformation	$\frac{al^2}{al^2 - ax'^2 + bx'^2}$	$\frac{2\theta_{xx}^2 (b-a) x' y'}{al^2}$	$\frac{1}{\theta_{xx}} + \frac{\theta_{xx}^2}{\theta_{xy}}$

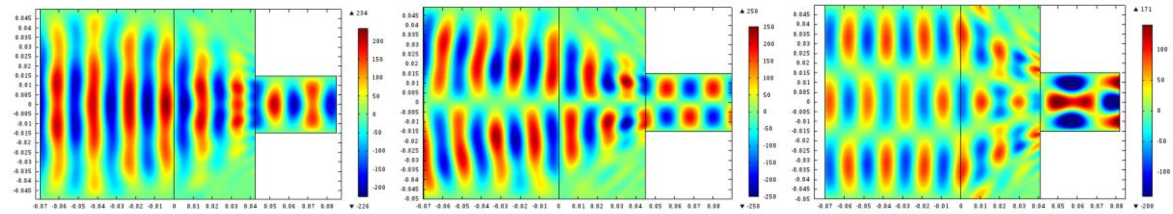

Figure 4: Adaptation of TE_1, TE_2 and TE_3 modes: linear transformation.

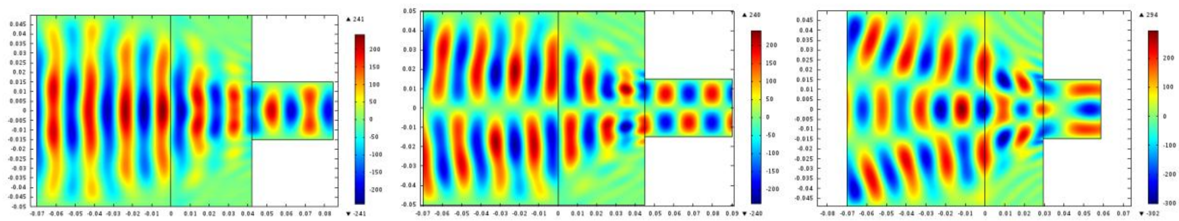

Figure 5: Adaptation of TE_1, TE_2 and TE_3 modes: exponential transformation.

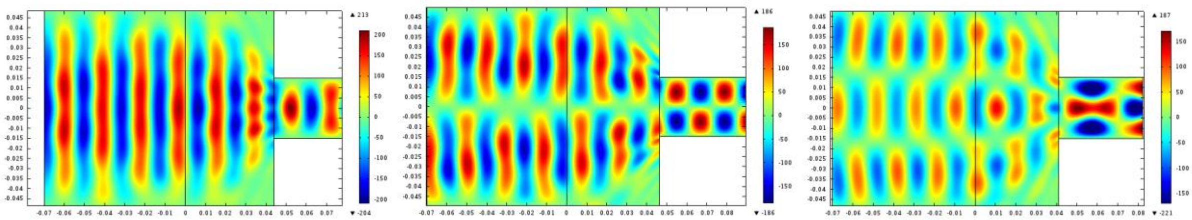

Figure 6: Adaptation of TE_1, TE_2 and TE_3 modes: parabolic transformation.

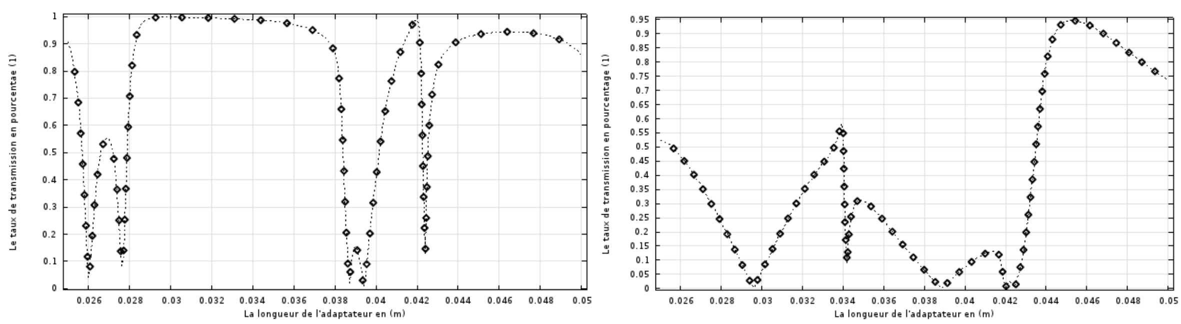

Figure 7: The transmission spectrum of TE_1 and TE_3 modes according to a conventional taper length

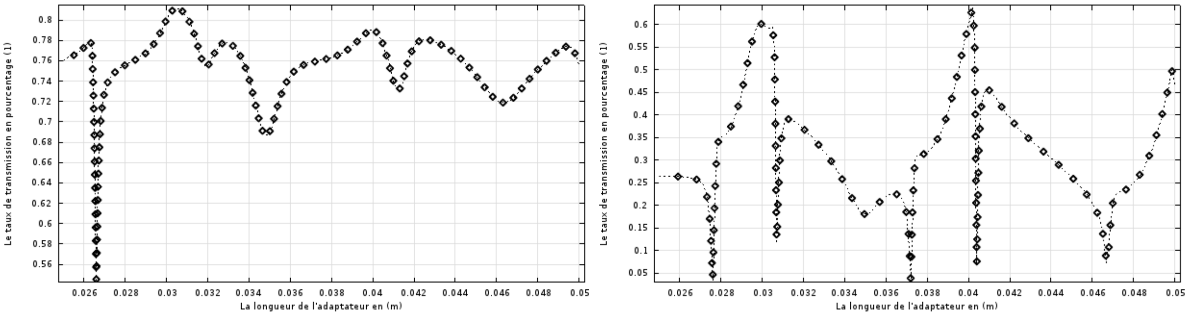

Figure 8: The transmission spectrum of TE_1 and TE_3 modes according to a linear transformation taper length

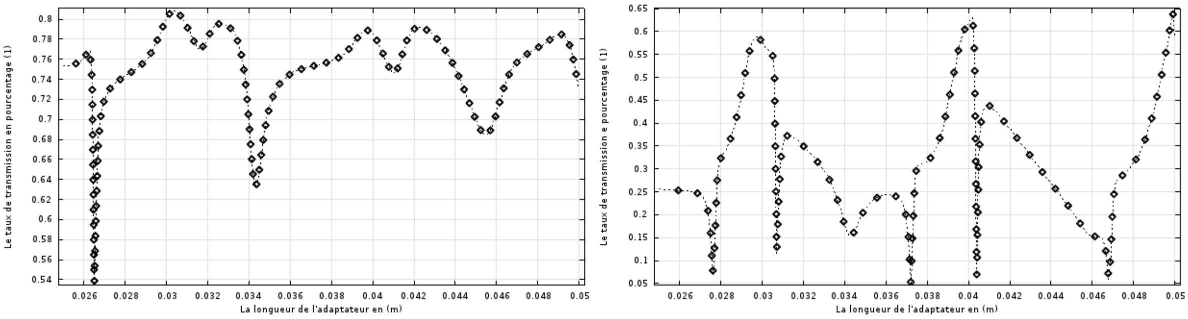

Figure 9: The transmission spectrum of TE_1 and TE_3 modes according to exponential transformation taper length

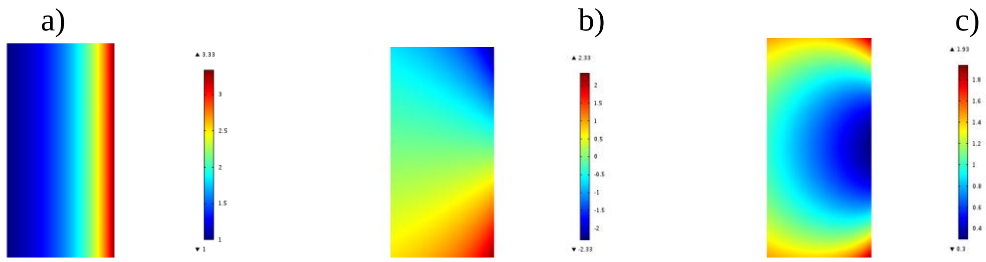

Figure 10: Spatial distribution of the components of permeability and permittivity tensors: linear transformation, a): $\theta_{xx} = \theta_{zz}$, b): θ_{xy} et c): θ_{yy}

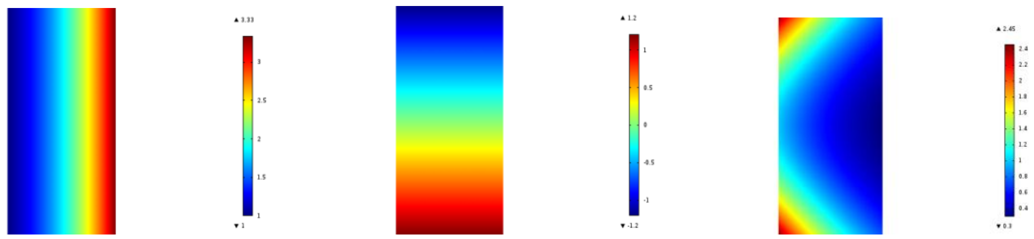

Figure 11: Spatial distribution of the components of permeability and permittivity tensors: exponential transformation, a): $\theta_{xx} = \theta_{zz}$, b): θ_{xy} et c): θ_{yy}

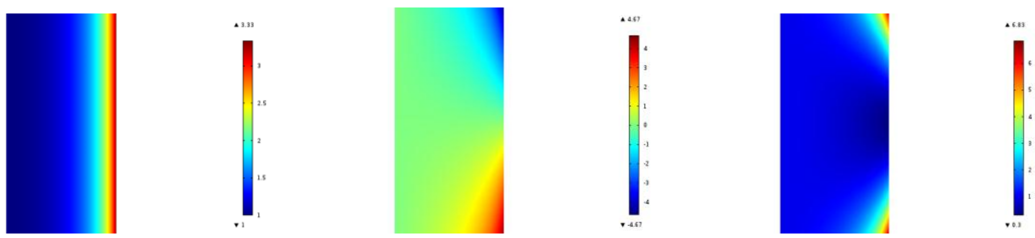

Figure 12: Spatial distribution of the components of permeability and permittivity tensors: parabolic transformation, a): $\theta_{xx} = \theta_{zz}$, b): θ_{xy} et c): θ_{yy}

one. A comparison of the transmission spectrum between a conventional adaptation process (Figure 7) and another based on linear compression (Figure 8) and exponential (Figure 9) spatial coordinates, for the TE_1 and TE_3 modes is realized: Concerning the conventional adaptation: when only the TE1 mode is excited, the electromagnetic energy is distributed mainly around the symmetry axis of the waveguide. That is why there is a strong signal transmission in the vicinity of 98% over a great length of the adapter from 2.9 to 3.7 cm. This transmission drops below 90% for the length that lies between 3.8 cm and 4.4 cm, which characterizes a reflection area. Meanwhile the transmission resumes emphasis from 4.4 cm to 5 cm (Figure 7- a). On the other hand, the TE3 mode is not properly guided. It appears from the low rate of transmission that exceeds the 90% only for a taper length between 4.42 cm and 4.63 cm. For the rest of lengths, the reflection phenomenon is preponderant (Figure 7-b).

For a metamaterials adaptation, the TE1 mode transmission rate is less important, it is just over 70% for most of the length of the taper, starting with the smallest length 2.5 cm and this for the two types of processing (figures 8-a, 9-a). This behavior is expected since the electromagnetic energy is concentrated in the vicinity of the optical axis, and the losses are minimal. While for the TE_3 mode, performance is poor. Indeed, the transmission rate is generally low when excluding the lengths 3 cm and 4 cm for the two types of transformations where the transmission is in the vicinity of 60%, and the length of 5 cm for the exponential transformation with a yield of 65% (figures 8-b, 9-b).

Based on this analysis, the two types of adapters have roughly the same behavior. The first mode is relatively well guided, while the higher modes are not. A comparison of these tapers with conventional ones gives a slight advantage to the later in terms of coupling rate of around 90% for the first mode, while it is around 70% for the proposed tapers. However, if we are interested to more compact components, metamaterial tapers become interesting especially for higher modes. Thus, for the TE3 mode, we obtain coupling rate close 60% to a length of 3 cm for both types of transformations. While one does not reach this value for a length of 4.4 cm with the conventional taper. Under these conditions, the metamaterial tapers can compete with the conventional ones in terms of compactness of the component.

Other hand, the spatial distribution of permeability and permittivity tensors shows that these values are realistic (figures 10-12). Therefore, their fabrication process of these components should not present major difficulties.

IV. Conclusion

We have shown the possibility of synthesis of an important component in integrated optics and telecommunication area: the taper based on metamaterials by exploiting the Cartesians coordinates transformation technic. Although the coupling rate between the input waveguide and the output waveguide remains relatively low, we can take advantage of the component's compactness if we are not very interested in a high transmission rate, especially for higher order modes.

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