Trajectory Tracking of Complex Dynamical Systems Using Delayed Recurrent Neural Networks Via PID Control Law

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***Abstract*— In this paper the problem of trajectory tracking is studied. Based on the Lyapunov-Krasovskii theory, a PID control law that achieves the global asymptotic stability of the tracking error between a delayed recurrent neural network and a complex dynamical network is obtained. To illustrate the analytic results we present a tracking simulation of a dynamical network with each node being just one Lorenz´s dynamical system and three identical Chen's dynamical systems.**

I. Introduction

This paper analyzes trajectory tracking not for a nonlinear system but for a network of coupled nonlinear systems with delay, which are forced to follow a reference signal generated by a nonlinear chaotic system. The control law that guarantees trajectory tracking is obtained by using the Lyapunov-Krasovskii methodology and the PID Control Law.

In previous chapters, we occasionally discussed the basic PID controllers[16]. For example, we presented electronic, hydraulic, and pneumatic PID controllers. We also designed control systems where PID controllers were involved. It is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

The proportional action tends to stabilize the system, while the integral control action tends to eliminate or reduce steady-state error in response to various inputs. Derivative Control Action. Derivative control action, when added to a proportional controller, provides a means of obtaining a controller with high sensitivity. An advantage of using derivative control action is that it responds to the rate of change of the actuating error and can produce a significant correction before the magnitude of the actuating error becomes too large. Derivative control thus anticipates the actuating error, initiates an early corrective action, and tends to increase the stability of the system.

The combination of proportional control action, integral control action, and derivative control action is termed proportional-plus-integral-plus-derivative control action. It has the advantages of each of the three individual control actions. The equation of a controller with this combined action is given by:

$$u_{ni} = K_{pi} e_i + K_{Vi} \dot{e}_i + K_i \int_0^t e_i(\tau)\, d\tau.$$

The analysis and control of complex behavior in complex networks, which consist of dynamical nodes, has become a point of great interest in recent studies, [1],[2],[3]. The complexity in networks comes from their structure and dynamics but also from their topology, which often affects their function.

Recurrent neural networks have been widely used in the fields of optimization, pattern recognition, signal processing and control systems, among others. They have to be designed in such a way that there is one equilibrium point that is globally asymptotically stable. In biological and artificial neural networks, time delays arise in the processing of information storage and transmission. Also, it is known that these delays can create oscillatory or even unstable trajectories, [4]. Trajectory tracking is a very interesting problem in the field of theory of systems control; it allows the implementation of important tasks for automatic control such as: high speed target recognition and tracking, real-time visual inspection, and recognition of context sensitive and moving scenes, among others. We present the results of the design of a control law that guarantees the tracking of general complex dynamical networks.

II. Mathematical models

1. General complex dynamical networks

Consider a network consisting of N linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system, described by

$$\dot{x}_i = f_i(x_i) + \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij} a_{ij}\, \Gamma\left(x_j - x_i\right), i = 1, \dots, N \qquad \text{Eq. 1}$$

where $x_i = (x_{i1}, \dots, x_{in})^T \epsilon\, \mathbb{R}^n$ are the state vectors of the node $i, f_i: \mathbb{R}^n \to \mathbb{R}^n$ represents the self-dynamics of the node i, the constants $c_{ij} > 0$ are the coupling strengths between node i and node j, with $i, j = 1, \dots, N, \Gamma = \left(\tau_{ij}\right) \epsilon\, \mathbb{R}^{nxn}$ is a constant internal matrix that describes the way of linking the components in each pair of connected node vectors $(x_j - x_i)$: this means that for some pairs (i, j) with $1 \leq i, j \leq n$ and $\left(\tau_{ij}\right) \neq 0$ the two coupled nodes are linked through their ith and jth sub-state variables respectively, while the coupling matrix $A = \left(\tau_{ij}\right) \epsilon\, \mathbb{R}^{NxN}$ denotes the coupling configuration of the entire network: this means that if there is a connection between node $i$and node j $i \neq j$, then $a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = a_{ji} = 0$.

III. Delayed recurrent neural networks

Consider a delayed recurrent neural network in the following form:

$$\dot{x}n_i = A_{n_i}x_{n_i} + W_{n_i}\sigma\left(x_{n_i}(t-\tau)\right) + u_{n_i} +$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,\Gamma(x_{nj}-x_{ni}),\ i=1,\dots,N, \qquad \text{Eq. 2}$$

where τ is the fixed known time delay ([5],[6]), $x_{ni} = (x_{ni1},\dots,x_{nin})^T \epsilon\, \mathbb{R}^n$ is the state vector of the neural network i, $u_{ni}\,\epsilon\,\mathbb{R}^n$ is the input of the neural network i, $A_{n_i} = -\lambda_{n_i}I_{nxn}$, $i=1,\dots,N$, is the state feedback matrix, with λ_{ni} being a positive constant, $W_{n_i}\,\epsilon\,\mathbb{R}^{NxN}$ is the connection weight matrix with $i=1,\dots,N$ 2, and $\sigma(\cdot)\,\epsilon\,\mathbb{R}^n$ is a Lipschitz sigmoid vector function ([7],[8]), such that $\sigma(x_{n_i})=0$ only at $(x_{n_i})=0$, with Lipschitz constant $L_{\sigma i}$ $i=1,\dots,N$ and neuron activation functions $\sigma_i(\cdot)=tanh(\cdot)$, $i=1,\dots,n$.

IV. TRAJECTORY TRACKING

The objective is to develop a control law such that the ith neural network (2) tracks the trajectory of the ith dynamical system (1).

We define the tracking error as $e_i = x_{ni}-x_i$, $i=1,\dots,N$ whose derivative with respect to time is

$$\dot{e}_i = \dot{x_{ni}} - \dot{x}_i,\quad i=1,\dots,N \qquad \text{Eq. 3}$$

Substituting (1) and (2) in (3), we obtain

$$\dot{e}_i = A_{n_i}x_{n_i} + W_{n_i}\sigma\left(x_{n_i}(t-\tau)\right) + u_{n_i} +$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,\Gamma(x_{nj}-x_{ni}) -$$
$$-f_i(x_i) - \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(x_j-x_i) \qquad \text{Eq. 4}$$

Adding and subtracting $W_{n_i}\sigma\left(x_{n_i}(t-\tau)\right)$, $\alpha_i(t)$, $i=1,\dots,N$, to (4), where α_i will be determined below, and considering that $x_{ni}=e_i+x_i$, $i=1,\dots,N$, then

$$\dot{e}_i = W_{n_i}(\sigma\left(x_{n_i}(t-\tau)\right) - \sigma(x_i(t-\tau))) +$$
$$+\left(u_{n_i}-\alpha_i(t)\right) + A_{n_i}e_i +$$
$$+(A_{n_i}x_i + W_{n_i}\left(\sigma(x_i(t-\tau)) + \alpha_i(t)\right) - f_i(x_i) +$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,\Gamma(x_{nj}-x_{ni}) - \qquad \text{Eq. 5}$$
$$-\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(x_j-x_i), i=1,\dots,N$$

In order to guarantee that the ith neural network (2) tracks the ith reference trajectory (1), the following assumption has to be satisfied:

Assumption 1. There exist functions $\rho_i(t)$ and $\alpha_i(t)$, $i=1,\dots,N$, such that

$$\frac{d\rho_i}{dt} = A_{n_i}\rho_i(t) + W_{n_i}\sigma(\rho_i(t)) + \alpha_i(t)$$
$$\rho_i(t) = x_i(t), i=1,\dots,N \qquad \text{Eq. 6}$$

Let's define

$$\widetilde{u_{ni}} = (u_{n_i} - \alpha_i(t))$$
$$\phi_\sigma(t-\tau) = \sigma\left(x_{n_i}(t-\tau)\right) - \sigma(x_i(t-\tau)) \qquad \text{Eq. 7}$$
$$i=1,\dots,N$$

Considering (6) and (7), equation (5) is reduced to

$$\dot{e}_i = A_{n_i}e_i + W_{n_i}\phi_\sigma(t-\tau) + \widetilde{u_{ni}} +$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,\Gamma(x_{nj}-x_{ni}) - \qquad \text{Eq. 8}$$
$$-\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(x_j-x_i),\qquad i=1,\dots,N$$

Writing the summations as

$$\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,\Gamma(x_{nj}-x_{ni}) =$$
$$=\Gamma\Big(\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\,x_{nj} - x_{ni}\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ninj}a_{ninj}\Big)$$

$$\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(x_j-x_i) = \qquad \text{Eq. 9}$$
$$=\Gamma\Big(\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,x_j - x_i\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\Big), i=1,\dots,N$$

and using that $c_{ninj}=c_{ij}$ and $a_{ninj}=a_{ij}$, then, using the equations above, (8) becomes

$$\dot{e}_i = A_{n_i}e_i + W_{n_i}\phi_\sigma(t-\tau) + \widetilde{u_{ni}} +$$
$$\Gamma\left(\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,e_j - e_i\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\right) = \qquad \text{Eq. 10}$$
$$= A_{n_i}e_i + W_{n_i}\phi_\sigma(t-\tau) + \widetilde{u_{ni}} +$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_j-e_i), i=1,\dots,N$$

It is clear that e_i, $i=1,\dots,N$ is an equilibrium point of (10), when $\widetilde{u_{ni}}$, $i=1,\dots,N$. In this way, the tracking problem can be restated as a global asymptotic stabilization problem for the system (10).

V. TRACKING ERROR STABILIZATION AND CONTROL DESIGN

In order to establish the convergence of (10) to $e_i=0$, $i=1,\dots,N$, which ensures the desired tracking, first, we propose the following Lyapunov function

$$V_N(e) = \sum_{i=1}^{N} V(e_i) =$$
$$\sum_{i=1}^{N}\big[\tfrac{1}{2}(e_i{}^T, w_i{}^T)(e_i, w_i)^{\ T} + \qquad \text{Eq. 11}$$

$$+\int_{t-\tau}^{t}(\phi_\sigma^T(s)W_{n_i}^T W_{n_i}\phi_\sigma(s))ds]$$

$$e=(e_1^T,\dots,e_N^T)^T$$

The time derivative of (11), along the trajectories of (10), and adding the Derivative "D"

$$v_N^*(e)=\sum_{i=1}^{N}[e_i^T\, e_i^* + w_i^T\, w_i^* + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau)] = \quad \text{Eq. 12}$$

$$=\sum_{i=1}^{N}[e_i^T\,(e_i^* + K_{V_i}e_i^*) + w_i^T\, w_i^* + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau)] =$$

$$=\sum_{i=1}^{N}[e_i^T\,(1+K_{V_i})e_i^* + w_i^T\, w_i^* + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau)]$$

If $a=(1+K_{V_i})$ and $w_i = K_{I_i}\int_0^t e_i(\tau)d\tau$, then $w_i^* = K_{I_i}e_i(t)$

$$v_N^*(e)=\sum_{i=1}^{N}[ae_i^T\,(A_{n_i}e_i + W_{n_i}\phi_\sigma(t-\tau) + \widetilde{u_{ni}} +$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_j-e_i)) + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) + w_i^T K_{I_i}e_i(t)]$$

Reformulating (12), we get

$$v_N^*(e)=\sum_{i=1}^{N}[-a\lambda_{n_i}\|e_i\|^2 + ae_i^T\, W_{n_i}\phi_\sigma(t-\tau) +$$

$$ae_i^T\widetilde{u_{ni}} + a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\, e_i^T\,\Gamma(e_j-e_i) + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) + w_i^T K_{I_i}e_i(t)] \quad \text{Eq. 13}$$

Next, let's consider the following inequality, proved in [9],[10]:

$$X^TY + Y^TX \le X^T\Lambda X + Y^T\Lambda^{-1}Y, \quad \text{Eq. 14}$$

which holds for all matrices X,Y ∈ $\mathbb{R}^{nxk}$ and $\Lambda \in \mathbb{R}^{nxn}$ with $\Lambda = \Lambda^T > 0$. Applying (14) with $\Lambda = I_{nxn}$ to the term

$e_i^T\, W_{n_i}\phi_\sigma(t-\tau),\ i=1,\dots,N$ we get

$$e_i^T\, W_{n_i}\phi_\sigma(t-\tau) \le$$

$$\le \tfrac{1}{2}e_i^Te_i + \tfrac{1}{2}\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) = \quad \text{Eq. 15}$$

$$=\frac{1}{2}\|e_i\|^2 + \frac{1}{2}\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau)$$

$$i=1,\dots,N$$

Then we have that

$$v_N^*(e) \le \sum_{i=1}^{N}[-a\lambda_{n_i}\|e_i\|^2 + \frac{a}{2}\|e_i\|^2 +$$

$$\frac{a}{2}\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) + ae_i^T\widetilde{u_{ni}} +$$

$$+a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j-e_i)) + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) -$$

$$-\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) + w_i^T K_{I_i}e_i(t)] \quad \text{Eq. 16}$$

By simplifying (16), we obtain

$$v_N^*(e) \le \sum_{i=1}^{N}[-a\lambda_{n_i}\|e_i\|^2 + \frac{a}{2}\|e_i\|^2 +$$

$$\left(\frac{a}{2}-1\right)\phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau) + ae_i^T\widetilde{u_{ni}} +$$

$$+a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j-e_i) + \quad \text{Eq. 17}$$

$$+ w_i^T K_{I_i}e_i(t) + \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t)] \le$$

$$\le \sum_{i=1}^{N}[-a\lambda_{n_i}\|e_i\|^2 + \tfrac{a}{2}\|e_i\|^2 + w_i^T K_{I_i}e_i(t) +$$

$$+\tfrac{a}{2}\phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) + ae_i^T\widetilde{u_{ni}} +$$

$$+a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j-e_i))$$

Since ϕ_σ is Lipschitz with Lipschitz constant $L_{\phi_{\sigma_i}}$ [7], then

$$\|\phi_\sigma(t)\| = \left\|\sigma\left(x_{n_i}(t)\right) - \sigma(x_i(t))\right\| \le$$

$$\le L_{\phi_{\sigma_i}}\left\|x_{n_i}(t) - x_i(t)\right\| = \quad \text{Eq. 18}$$

$$= L_{\phi_{\sigma_i}}\|e_i(t)\|, \qquad i=1,\dots,N$$

Applying (18) to $\phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t)$ we obtain

$$\phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) \le \left\|\phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t)\right\| \le$$

$$\le (L_{\phi_{\sigma_i}})^2\left\|W_{n_i}\right\|^2\|e_i\|^2,\ i=1,\dots,N \quad \text{Eq. 19}$$

Now (17) is reduced to

$$v_N^*(e) \le \sum_{i=1}^{N}[-a\lambda_{n_i}\|e_i\|^2 + \tfrac{a}{2}\|e_i\|^2 +$$

$$+\tfrac{a}{2}(L_{\phi_{\sigma_i}})^2\left\|W_{n_i}\right\|^2\|e_i\|^2 + ae_i^T\widetilde{u_{ni}} + \quad \text{Eq. 20}$$

$$+a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j-e_i) + w_i^T K_{I_i}e_i(t)] =$$

$$=\sum_{i=1}^{N} e_i^T\,(-a\,\lambda_{n_i}e_i - a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_i) +$$

$$+\frac{a}{2}(1+\left(L_{\phi_{\sigma_i}}\right)^2\left\|W_{n_i}\right\|^2)e_i + w_i^T K_{I_i}e_i(t) +$$

$$+ a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j) + a\widetilde{u_{ni}})$$

We define

$$\widetilde{u_{ni}} = \widetilde{u_{ni}}^{(1)} + \widetilde{u_{ni}}^{(2)} + K_{p_i}e_i + w_i - \\ +\frac{\gamma}{2}(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2)$$

Where $w_i = K_{i_i}\int_0^t e_i(\tau)d\tau,\ i = 1, \dots, N$, and then (20) becomes

$$v_N^*(e) \le \sum_{i=1}^{N} e_i^T\,(-a\,\lambda_{n_i}e_i - a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_i) + \\ +\frac{a}{2}(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2)e_i + a\widetilde{u_{ni}}^{(1)} + a\,K_{p_i}e_i + \\ + a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^T\,\Gamma(e_j) + a\widetilde{u_{ni}}^{(2)} + aw_i - \\ -\frac{a\gamma}{2}\left(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2\right) + w_i^T K_{i_i}e_i(t))$$

Then,

$$v_N^*(e) \le \sum_{i=1}^{N}[-a(\lambda_{n_i} - K_{p_i})e_i^T e_i - \\ -\frac{a(\gamma-1)}{2}\left(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2\right)\, e_i^T e_i + \\ (a + K_{i_i})e_i^T w_i - \ a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma e_i^T e_i + \qquad \text{Eq. 21} \\ + a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma e_i^T(e_j) + ae_i^T\widetilde{u_{ni}}^{(1)} + ae_i^T\widetilde{u_{ni}}^{(2)}]$$

Here, we select $(a + K_{i_i}) = 0$, so $K_{V_i} = -K_{i_i} - 1$; $K_{V_i} \ge 0$,then $K_{i_i} \ge -1$. With this selection of parameters (21) is reduced to:

$$v_N^*(e) \le \sum_{i=1}^{N}[-a(\lambda_{n_i} - K_{p_i})e_i^T e_i - \\ -\frac{a(\gamma-1)}{2}\left(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2\right)\, e_i^T e_i - \\ a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma e_i^T e_i + \\ + a\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma e_i^T(e_j) + ae_i^T\widetilde{u_{ni}}^{(1)} + ae_i^T\widetilde{u_{ni}}^{(2)}]$$

In this part, if $\lambda_{p_i} - K_{p_i} > 0$, $a > 0$ and $\gamma - 1 > 0$, then $v_N^*(e) < 0,\ \forall\, e_i, w_i, W_{n_i} \neq 0$, the error tracking is asymptotically stable and it converges to zero for every $e_i \neq 0$, i.e. the Neural Network will follow the plant asymptotically.

Now, we propose to use the following control law:

$$\widetilde{u_{ni}} = (1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2)e_i - \\ -\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_j),\ \ i = 1, \dots, N \qquad \text{Eq. 22}$$

Then $v_N^*(e) < 0$ for all $e_i \neq 0$. This means that the proposed control law (22) can globally and asymptotically stabilize the ith error system (10), therefore ensuring the tracking of (1) by (2). Finally, the control action driving the recurrent neural networks is given by:

$$u_{ni} = f_i(x_i) + \lambda_{n_i}x_i - W_{n_i}(\sigma\left(x_{n_i}(t-\tau)\right) + \\ +(\frac{1}{2} + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2)e_i + \ K_{pi}e_i + K_{Vi}\dot{e_i} + K_i\int_0^t e_i(\tau)\,d\tau \\ -\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\,\Gamma(e_i),\ \ i = 1, \dots, N \qquad \text{Eq. 23}$$

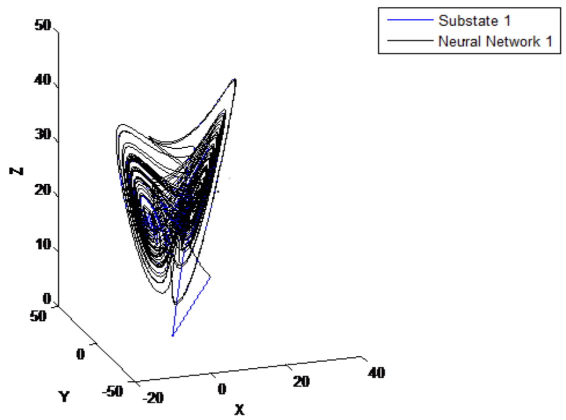

Fig. 1 Sub-State and Neural Network of Lorentz's attractor with initial condition $X_1(0) = (10;\ 0;\ 10)^T$

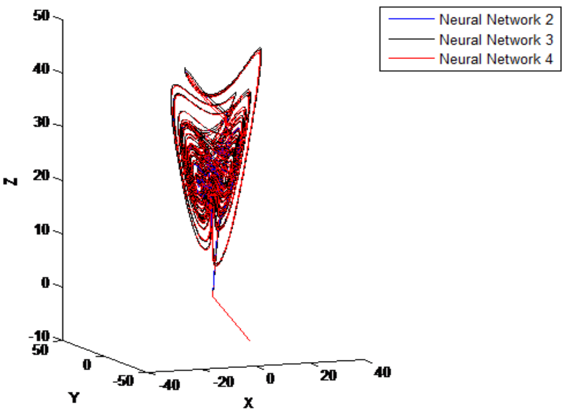

Fig. 2 Sub-States of Chen's attractor with initial condition $X_{2,3,4}(0) = (-10,0,37)^T$

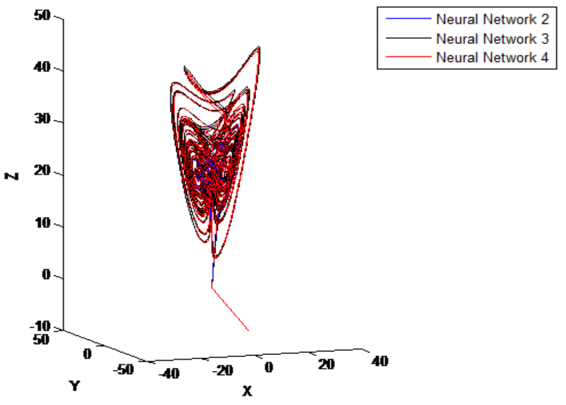

Fig. 2.1 Neural Network of Chen's attractor with initial condition $X_{2,3,4}(0) = (-10,0,37)^T$

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VI. SIMULATIONS

In order to illustrate the applicability of the discussed results, we consider a dynamical network with just one Lorenz's node and three identical Chen's nodes. The single Lorenz system is described by:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = \begin{pmatrix} 10x_2 - 10x_1 \\ -x_2 - x_1x_2 + 28x_1 \\ x_1x_2 - (\frac{8}{3})x_3 \end{pmatrix} \qquad \text{Eq. 24}$$

$$x_i(0) = (10,0,10)^T,\ i = 1$$

and the Chen's oscillator is described by

$$\begin{pmatrix} \dot{x_{i1}} \\ \dot{x_{i2}} \\ \dot{x_{i3}} \end{pmatrix} = \begin{pmatrix} p_1(x_{i2} - x_{i1}) + \sum_{j=1, j\neq i}^{4} c_{ij}a_{ij}(x_{j1} - x_{i1}) \\ (p_3 - p_2)x_{i1} - x_{i1}x_{i3} + p_3x_{i2} + \sum_{j=1, j\neq i}^{4} c_{ij}a_{ij}(x_{j2} - x_{i2}) \\ x_{i1}x_{i2} - p_2x_{i3} + \sum_{j=1, j\neq i}^{4} c_{ij}a_{ij}(x_{j3} - x_{i3}) \end{pmatrix}$$

$$x_i(0) = (-10,0,37)^T,\ i = 2,3,4 \qquad \text{Eq. 25}$$

If the system parameters are selected as $p_1 = 35$, $p_2 = 3$, $p_3 = 28$, then the Lorenz's system and Chen's system are shown in Fig. 1 and Fig.2 respectively. In this set of system parameters, one unstable equilibrium point of the oscillator (25) is $x = (7.9373,7.9373,21)^T$ [11].

Suppose that each pair of two connected Lorenz and Chen's oscillators are linked together through their identical sub-state variables, i.e., Γ=diag(1,1,1), and the coupling strengths are $c_{12} = c_{21} = \pi$, $c_{13} = c_{31} = \pi$, $c_{23} = c_{32} = \pi$, $c_{14} = c_{41} = 2\pi$, $c_{24} = c_{42} = 2\pi$, $c_{34} = c_{43} = 2\pi$. Fig. 3 visualizes this entire dynamical network:

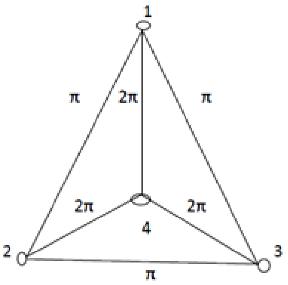

Fig. 3 Structure of the network with each node being a Lorentz and Chen's system.

The neural network is selected as:

$$A_{n_i} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad W_{n_i} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 4 & 0 \\ 0 & 2 & 3 \end{pmatrix},$$

$$\sigma\left(x_{n_i}(t-\tau)\right) = \begin{pmatrix} \tanh(x_{n_i1}(t-\tau)) \\ \tanh(x_{n_i2}(t-\tau)) \\ \tanh(x_{n_i3}(t-\tau)) \end{pmatrix}, \qquad \text{Eq.26}$$

τ = 10seconds τ = 10seconds,

$$\left(L_{\phi_{\sigma_i}}\right) \triangleq n = 3$$

$$x_{n_i}(0) = (20,20,-10)^T, \qquad i = 1,2,3,4$$

Theorem: For the unknown nonlinear system modeled by (1), the on-line learning law $tr\{W^TW\} = -e_i{}^T W\sigma(x)$ and the control law (23) ensure the tracking of nonlinear reference model (4), [13].

Remark: From (21) we have

$$v_N^*(e) \le \sum_{i=1}^{N}[-a(\lambda_{n_i} - K_{p_i})e_i^T e_i - \frac{a(\gamma-1)}{2}\left(1 + \left(L_{\phi_{\sigma_i}}\right)^2 \|W_{n_i}\|^2\right) e_i^T e_i + (a + K_{i_i})e_i^T w_i - a\sum_{j=1, j\neq i}^{N} c_{ij}a_{ij}\,\Gamma e_i^T e_i + a\sum_{j=1, j\neq i}^{N} c_{ij}a_{ij}\,\Gamma e_i^T(e_j) + ae_i^T\widetilde{u_{ni}}^{(1)} + ae_i^T\widetilde{u_{ni}}^{(2)}] < 0$$

$\forall\ e_i, w_i, W_{n_i} \neq 0$, and therefore V is decreasing and bounded from below by V(0). Since

$$V_N(e) = \sum_{i=1}^{N}[\tfrac{1}{2}(e_i{}^T, w_i{}^T)(e_i, w_i)^{\ T} + \int_{t-\tau}^{t}(\phi_\sigma^T(s)W_{n_i}^T W_{n_i}\phi_\sigma(s))ds]$$

then we conclude that $e_i, W_{n_i} \epsilon\ L_1$; this means that the weights remain bounded.

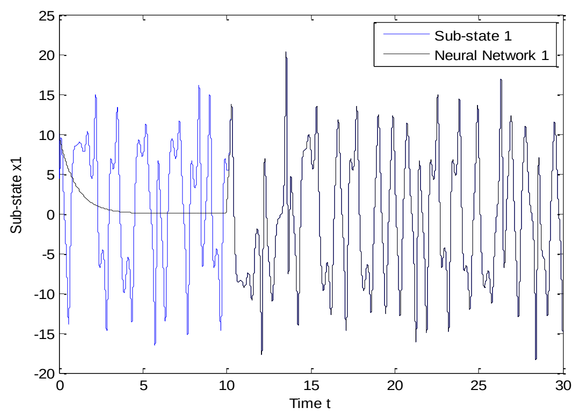

Fig. 4 Time evolution for sub-states 1 with initial state $X_{n_1}(0) = (10,0,10)^T$

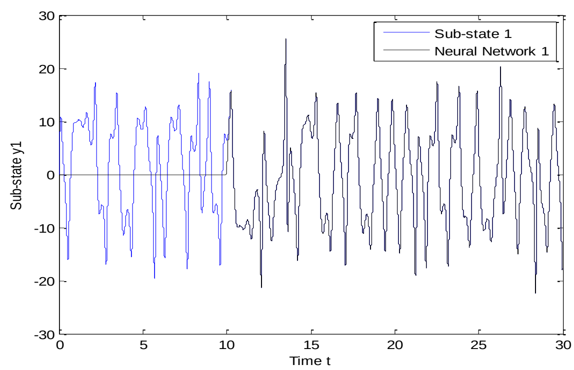

Fig. 5 Time evolution for sub-states 1 with initial state

$X_{n_1}(0) = (10,0,10)^T$

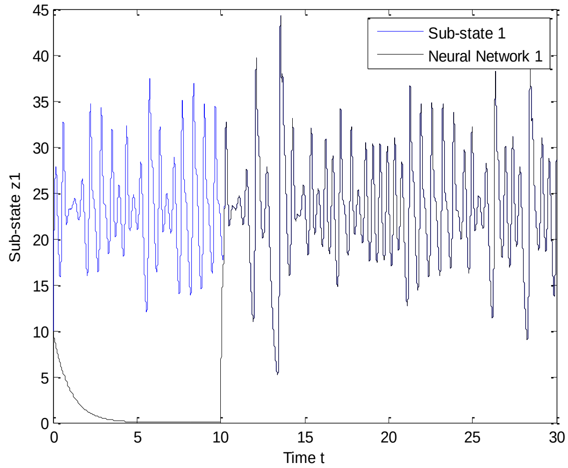

Fig. 6 Time evolution for sub-states 2 with initial state $X_{n_1}(0) = (10,0,10)^T$

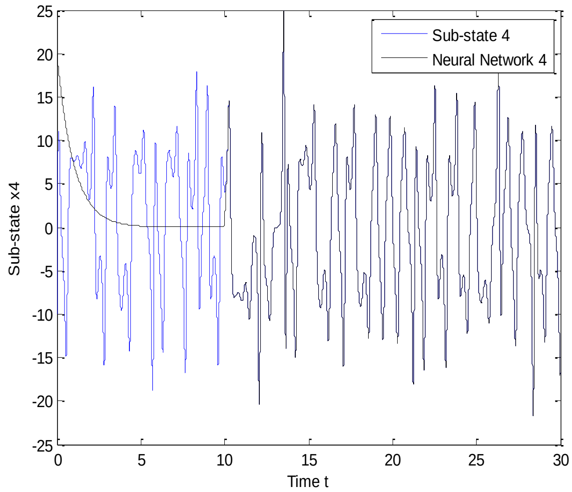

Fig. 7 Time evolution for sub-states 4 with initial state $X_{n_4}(0) = (20,20,-10)^T$

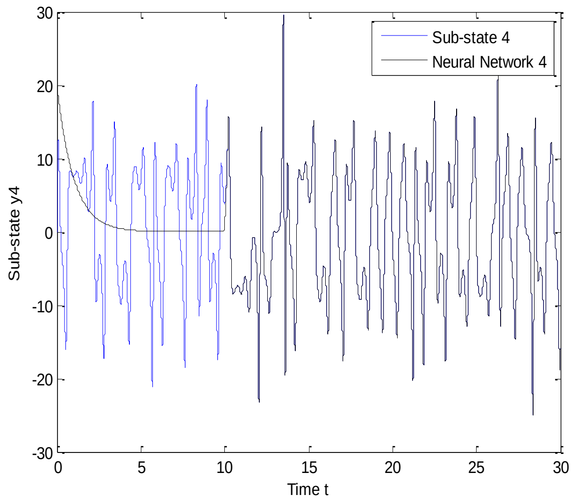

Fig. 8 Time evolution for sub-states 4 with initial state $X_{n_4}(0) = (20,20,-10)^T$

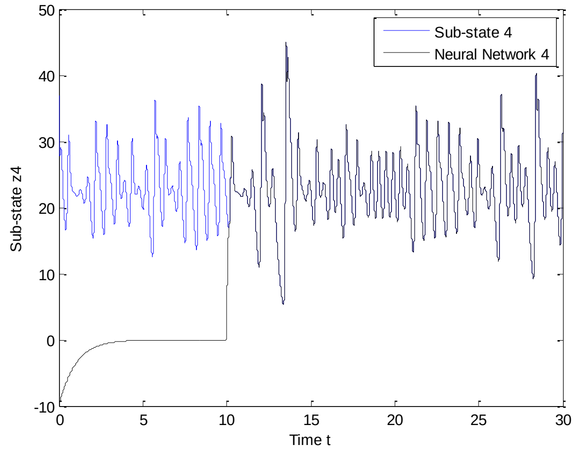

Fig. 9 Time evolution for sub-states 4 with initial state $X_{n_4}(0) = (20,20,-10)^T$

The experiment is performed as follows. Both systems, the delayed neural network (2) and the dynamical networks (24) and (25), evolve independently until t=10 seconds; at that time, the proposed control law (23) is incepted. Simulation results are presented in Fig. 4 - Fig. 6 for sub-sates of node 1. As can be seen, tracking is successfully achieved and error is asymptotically stable, as it is shown in Fig. 7 -Fig. 9 for sub-states of node 4.

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VII. CONCLUSIONS

We have presented the controller design for trajectory tracking determined by a general complex dynamical network. This framework is based on dynamic delayed neural networks and the methodology is based on Lyapunov-Krasovskii theory. The proposed PID Control Law [14] is applied to a dynamical network with each node being a Lorenz and Chen's dynamical system [15], respectively, being able to also stabilize in asymptotic form the tracking error between two systems. The results of the simulation shows clearly the desired tracking. In future work, we will consider the stochastic case for the complex dynamical network.

REFERENCES

[1] Wang, X. (2002). Complex networks: Topology, dynamics and synchronization, Int. J. Bifurcation Chaos, vol. 12, no. 5, pp. 885--916.
[2] Wu, C. (2002). Synchronization in Coupled Chaotic Circuits and Systems, World Scientific, Singapore.
[3] Newman, M. (2003). The structure and function of complex networks, SIAM Review 45, 167--256.
[4] Baldi, P. & A., A. (1994). How delays affect neural dynamics and learning, IEEE Trans. on Neural Networks, vol. 5, pp. 612--621.
[5] Liao, X., Chen, G. & Sanchez, E. (2002). Lmi-based approach for asymptotically stability analysis of delayed neural networks, IEEE Trans. on Circuits and Systems, vol. 49, no. 7, pp. 1033--1039.

[6] Liao, X., Chen, G. & Sanchez, E. (2002). Delay-dependent exponential stability analysis of delayed neural networks: an lmi approach, Neural Networks, vol. 15, pp. 855--866.

[7] Khalil, H. (1996). Nonlinear System Analysis, 2nd. Ed. Prentice Hall, Upper Saddle River, NJ, USA.

[8] Krstic, M. & Deng, H. (1998). Stabilization of Nonlinear Uncertain Systems, Springer Verlag, USA.

[9] Sanchez, E. & Perez, P. (1999). Input-to-state stability analysis for dynamic neural networks, IEEE Trans. on Circuits Syst. I, vol 46, pp. 1395--1398.

[10] Edgar N. Sanchez, Jose P. Perez, Joel Perez.(2006) Trajectory Tracking for Delayed Recurrent Neural Networks. American control Conference, vol. issue, 14-16

[11] Li, X., Wang, X. & Chen, G. (2004). Pinning a complex dynamical network to its equilibrium, IEEE Transactions on Circuits and Systems I: Regular Papers 51(10): 2074--2087.

[12] Rovitahkis G. A. and M. A. Christodoulou, Adaptive Control with Recurrent High-Order Neural Networks, Springer Verlang, New York, USA, 2000.

[13] Petros A. Ioannou, Jing Sun, "Robust Adaptive Control", PTR Prentice-Hall, Upper Saddle River, NJ 07458, ISBN 0-13-439100-4.

[14] Joel Perez P, Jose P. Perez, Jose J. Hdz, Santiago Arroyo, Angel Flores, "Trjectory Tracking Error Using PID Control Law for a 2 DOF Helicopter Model Via Adaptive Time-Delay Neural Networks", Congreso Nacional de Control Automatico 2013, Ensenada, Baja California, Mexico, Octubre 16-18, 2013.

[15] Jose P. Perez, Joel Perez Padron, Angel Flores Hernandez, Santiago Arroyo, " Complex Dynamical Network for Trajectory Tracking Using Delatey Recurrent Neural Networks", Hindawi Publishing Corporation, Mathematical Problems in Engineering, Volume 2013, Article ID 162610, http://dx.doi.org/2013/162610, Received 21 March 2013; Accepted 12 Septembes 2013.

[16] Katsuhiko Ogata, Modern Control Engineering, Fifth Edition, Copyright © 2010, 2002, 1997, 1990, 1970 Pearson Education, Inc., publishing as Prentice Hall, ISBN 10: 0-13-615673-8, ISBN 13: 978-0-13-615673-4