Identifying Structurally Identical Links in a Kinematic Chain

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***Abstract*— Decimal codes of kinematic chains are readily amenable to retrieval of parent chain through decoding. and, as such, they hold the key to enumeration process. An optimum selection of range of Decimal codes of chains of a given type, which permits all possible types of connectivity to the link with highest degree (i.e., with highest number of elements), can provide a computerized method of enumeration. The concept is also shown useful in identifying Structurally identical mechanisms. As these identification codes are decodable, they enable reconstruction of topology of kinematic chains/mechanisms on computer.**

***Index Terms*— Kinematic Chain, Adjacency Matrix, UTAM(Upper Triangular Adjacency Matrix), Decimal codes..**

I. Introduction

Graph theoretic representation of a chain, with a link represented by a vertex and a joint by an edge, leads to a link-link connectivity matrix (also called the zero-one adjacency matrix) for a labeled kinematic chain with simple joints. It turns out to be a powerful tool in structural analysis and is well suited to computer processing. Ambekar and Agrawal [1] proposed to represent a kinematic chain and mechanisms through a set of max codes. Also, Graph theoretic literature [2], shows that method of calculating the characteristic polynomial is neither simple nor does it provide any insight into the relation between the graph structure and different coefficients. Read and Corneil [3] remarked that a good solution to the coding problem provides a good solution to the isomorphism problem, though, the converse is not necessarily true. This goes to suggest that a suitable coding procedure can be relied upon for structural identification of chains and mechanisms.

II. Identification Codes and Canonical Numbers

A. A Brief Introduction.

A kinematic chain of n-links, can be represented by a link-link adjacency matrix for a given scheme of labeling and a binary code can be readily extracted from the Upper triangular adjacency matrix (abbreviated by UTAM). To establish a binary code, for the given scheme of labeling, one considers the UTAM, extracted from the link-link adjacency matrix of the kinematic chain. A binary sequence is established by laying strings of 'zeros ' and 'ones 'in rows '1_i' through ($i < 1$) of the UTAM, one after the other, in a sequence 'from top to bottom'. This binary sequence may be looked upon as a binary number/code. From the binary code, so obtained, a decimal number can be readily established giving an Identification code, for the given kinematic chain, which may not be unique. For an n-link kinematic chain, theoretically, a total of n! schemes of labeling are possible. The zero-one link-link adjacency matrix is a square symmetric matrix with a total of n^2 number of elements in it. Thus, accounting for 'n_i' number of diagonal elements, the total number of elements in UTAM are n(n-1)/2. Thus, the length (i.e. number of digits) of the binary sequence is also, n (n-1) / 2. It follows from preceding paragraph that a total of n! binary/decimal numbers are possible for the same chain. All the binary/decimal numbers, in general, are not distinct. When all the n! decimal numbers of an n-link kinematic chain are laid down in say, descending order, the greatest and the smallest one are easily recognized as the unique or canonical numbers for the given chain.

The canonical labeling scheme [4]for a kinematic chain, for which the decimal number is in some (either maximum or minimum) canonical form, is either a maxcode or mincode

Consider the Kinematic Chain as shown in Fig. 1. with 7 links : 6 binary and 1 quaternary links with two degree of freedom.

For the maxcode labeled graph as shown in Fig.2, the adjacency matrix and the corresponding UTAM (Upper Triangular Adjacency Matrix) is as under

$$\mathbf{A} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{vmatrix}$$

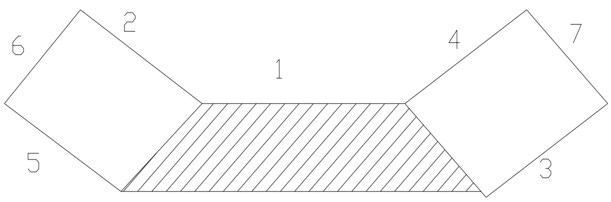

Fig. 1. Kinematic Chain of seven links

Manuscript received December 20, 2013.
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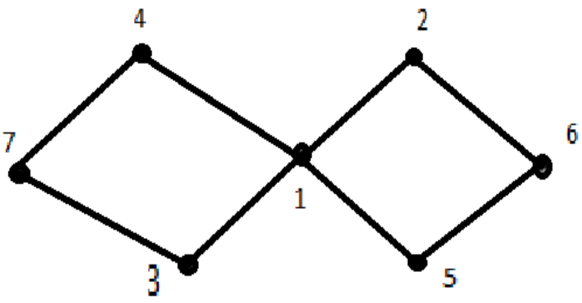

Fig.2.Graphical representation of seven links Chain

$$\text{UTAM} = \left|\begin{array}{cccccc} 1 & 1 & 1 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 \\ & & 0 & 0 & 0 & 1 \\ & & & 0 & 0 & 1 \\ & & & & 1 & 0 \\ & & & & & 0 \end{array}\right|$$

There are twenty one entries in the UTAM which, if written consecutively by rows as

111100 ; 00010 ; 0001 ; 001 ; 10 ; 0

results in a binary sequence = 111100000100001001100

One can look at the resulting strings of 'ones' and 'zeros' as representing digits a binary code. And then it is more convenient to express maxcode in corresponding decimal form as –

$M(G) = 1.(2^{20}) + 1.(2^{19}) + 1.(2^{18}) + 1.(2^{17}) + 0.(2^{16}) + 0.(2^{15}) + 0.(2^{14}) + 0.(2^{13}) + 0.(2^{12}) + 1.(2^{11}) + 0.(2^{10}) + 0.(2^{9}) + 0.(2^{8}) + 0.(2^{7}) + 1.(2^{6}) + 0.(2^{5}) + 0.(2^{4}) + 1.(2^{3}) + 1.(2^{2}) + 0.(2^{1}) + 0.(2^{0}) = 1968204$

It can be verified that the same chain, with changed labeling as at Fig.(3),has the same identification code.

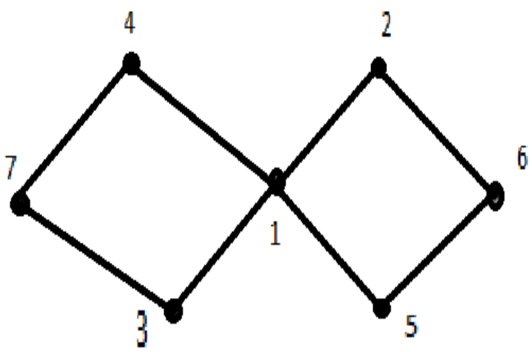

Fig. 3

Comparing the two chains at Fig. (2) and Fig. (3) it may be concluded that links 2 and 4 are structurally identical in combination with links 3 and 5 respectively. Thus link pairs 2 and 4 can be exchanged with link pairs 3 and 5 mutually. As against this, when link labels 2 and 4; 3 and 5; 6 and 7 are exchanged in isolation, the UTAM and the binary code/ decimal code remain unchanged. The links 2 and 4; 3 and 5; 6 and 7 are therefore called structurally identical links. The numbers in the bracket, as shown at Fig.(4), indicate the alternative scheme of labeling which yields the same Identification code.

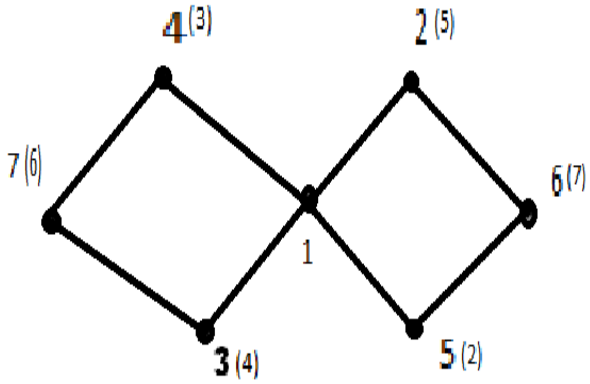

Fig. 4

III. Conclusion:

The concept of identification code is shown to be useful in identifying structurally identical links or pair of links in a chain, which in turn, helps in identifying structurally equivalent mechanisms with an example of 7 link 2 DOF kinematic chain. The concept of identification codes is claimed to be important from the point of view of a computerized method of enumeration of kinematic chains. The concept identification codes of mechanisms, when extended, is likely to be of great help in developing a system of mechanisms classification and indexing.

References

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