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Canonical numbering of seven link Kinematic Chain

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Abstract— **The objective is to know the significance of canonical numbering along with the application. Each kinematic chain with an identification code either a maxcode, or mincode which is also known as canonical numbering is being a unique technique of representing Kinematic chains. In the proposed paper the identification code i.e. maxcode and mincode is explained along with an example of seven links 2 degree of freedom (DOF) kinematic Chain.**

Index Terms— **Kinematic Chain, Adjacency Matrix, UTAM(Upper Triangular Adjacency Matrix), Maxcode, Mincode.**

I. Introduction

Read and Corneil [1] remark that a good solution to the coding problem provides a good solution to the isomorphism problem , though, the converse is not necessarily true. This goes to suggest that a successful solution to the isomorphism problem can be obtained through coding. The concept of canonical numbers provide identification codes which are unique for structurally distinct kinematic chains. One important feature of canonical numbers is that they are decodable, and also promises a potentially powerful method of identifying structurally equivalent links and pairs in a kinematic chain. While describing a method of storing the details of adjacency matrix in a binary sequence, maxcode and mincode are the tools for the identification of kinematic chains. The test of isomorphism then reduces to the problem of comparing max/mincodes of the two chains.

II. Canonical Numbering

According to Ambekar and Agrawal [2] the concepts of maxcode and mincode were introduced as canonical number for the enumeration. For every kinematic chain of n-links, there are n! different ways of labeling the links and hence, n! different binary numbers are possible for the same chain. By arranging these n! binary numbers in an ascending order, two extreme binary numbers can be identified , as significant ones for the same chain. These two binary numbers are : the maximum number and minimum number designated , respectively as maxcode M(K) and mincode m(K) . Since M(K) and m(K) denotes two extreme values of binary numbers for a given kinematic chain , and each has a unique position in the hierarchical order and is easily recognized, they are called as canonical numbers. Again each binary number of a given kinematic chain corresponds to a particular adjacency matrix , and hence it also corresponds to a particular labeling scheme.

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The unique labeling scheme for the links of a kinematic chain, for which the binary number is in some (either maximum or minimum) canonical form, is called *canonical numbering* (labeling) of the chain. The adjacency matrix, which corresponds to canonical numbering, is said to be in some canonical form.

III. Property of Canonical Numbering

According to [3]In a binary number an entry of '1' as an (i+1)th digit, counted from the right hand end , has a contribution to the decimal code equal to 2^i . Also, it follows from the basic property of binary numbers that,

$$2^i > 2^0 + 2^1 + 2^2 + 2^3 + \ldots\ldots + 2^{(i-1)}$$

This is obvious because the right hand side of the above inequality represents summation of terms in geometrical progression and hence, can be shown to be equal to $(2^i - 1)$. This goes to prove that a contribution of any '1', in a binary number , is more significant than even the joint contribution of all the subsequent 'ones' in that binary number. This is fundamental to a basic understanding of any algorithm on maxcode and mincode.

For the purpose of establishing a binary code one considers upper triangular adjacency matrix for the canonical labeling of the chain. Binary sequence is established by laying strings of zeros and ones in rows, '1' through (i – 1) row, one after the other in a sequence from top to bottom. This binary sequence may be regarded as a binary number illustrated by an example: Consider the Kinematic Chain as shown in Fig. 1. with 7 links : 6 binary and 1 quaternary links with two degree of freedom.

IV. Application of Canonical Numbering with Decodability

According to Bauchabaum and Freudenstein [4] who gave the graphical method to represent kinematic structure which consists of polygons and lines representing links of different degrees, connected by small circles representing pairs/joints. It is the powerful tool as it is well suited to computer implementation , by using adjacency matrices to represent the graph. The graphical representation of the same chain can be explained in Fig. 2.

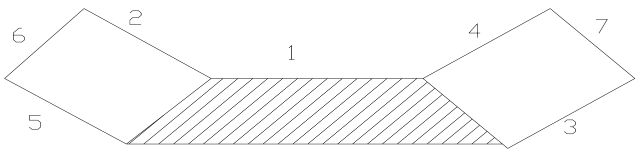

Fig. 1. Kinematic Chain of seven links

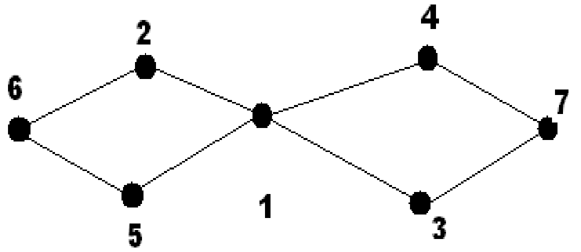

Fig.2.Graphical representation of seven links Chain

For the maxcode labeled graph as shown in Fig.2, maxcode the adjacency matrix and the corresponding UTAM (Upper Triangular Adjacency Matrix) is as under

$$A = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{vmatrix}$$

$$\text{UTAM} = \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 \\ & & 0 & 0 & 0 & 1 \\ & & & 0 & 0 & 1 \\ & & & & 1 & 0 \\ & & & & & 0 \end{vmatrix}$$

There are twenty one entries in the UTAM which, if written consecutively by rows as 111100 ; 00010 ; 0001 ; 001 ; 10 ; 0
results in a binary sequence = 111100000100001001100

One can look at the resulting strings of 'ones' and 'zeros' as representing digits a binary code. And then it is more convenient to express maxcode in corresponding decimal form as –

$M(G) = 1.(2^{20}) +1.(2^{19}) + 1.(2^{18}) + 1.(2^{17}) + 0.(2^{16})+ 0.(2^{15}) +0.(2^{14}) +0.(2^{13})+0.(2^{12})+1.(2^{11}) +0.(2^{10})+0.(2^{9}) +0.(2^{8}) +0.(2^{7}) +1.(2^{6}) +0.(2^{5}) + 0.(2^{4}) +1.(2^{3}) +1.(2^{2}) +0.(2^{1}) +0.(2^{0}) = 1968204$

Intuitively, the labeling of the same graph can be as at Fig.3. giving the min code m(G) and the corresponding UTAM will be

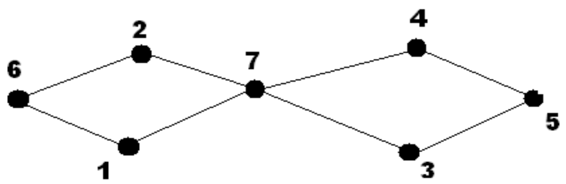

Fig.3.Graph with mincode labeling

$$\text{UTAM} = \begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ & 0 & 0 & 0 & 1 & 1 \\ & & 0 & 1 & 0 & 1 \\ & & & 1 & 0 & 1 \\ & & & & 0 & 0 \\ & & & & & 0 \end{vmatrix}$$

At Fig. 3, for min code m(G), there are twenty one entries in the UTAM which, if written consecutively by rows as 000011 ; 00011 ; 0101 ; 101 ; 00; 0 , the corresponding binary sequence is

m(G) = 000011000110101101000

And the corresponding decimal min code is

$m(G) = 0.(2^{20}) + 0.(2^{19}) + 0.(2^{18}) + 0.(2^{17}) + 1.(2^{16}) + 1.(2^{15}) +0.(2^{14}) + 0.(2^{13})+0.(2^{12})+1.(2^{11}) +1.(2^{10})+0.(2^{9}) +1.(2^{8}) +0.(2^{7}) +1.(2^{6}) +1.(2^{5}) + 0.(2^{4}) +1.(2^{3}) +0.(2^{2}) +0.(2^{1}) +0.(2^{0}) = 101736$

A. Decodability:- For the given identification code, it is possible to reconstruct the linkage topology on the basis of these canonical numbers alone. This is made possible by the division of the identification code by 2. The reminders are arranged sequentially to get again the binary number with which the linkage topology can be reconstructed.

V. Conclusion

This paper demonstrates the power and potential of canonical number, with an application of the same to 7 link 2 DOF Kinematic Chain. The scheme of number representation gives unique canonical labeling and codes for all the chains and mechanisms and are decodable. Thus, the canonical numbering (either maxcode or mincode) : being unique and decodable holds great promise in cataloguing (storage and retrieval) of kinematic chains and mechanisms.

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