Natural Number series from Prime Number

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Abstract— **In this paper we will show how natural number series can be formed by prime numbers when 1,2 is given and operator multiplication is given. We will then prove Legendre and Grimm's Conjecture.**

Index Terms— **prime numbers, Grimm's Conjecture.**

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II. What is prime

A number which is divisible by 1 and that number only is called a prime number.

I suggest another definition of prime is : When we adorn odd numbers in increasing series in natural number line starting with 1,2,3 there is gap of odd numbers. This gap numbers are filled with prime numbers. See the example below under "How primes generate"

All odd composite number can be written in the form : ΠΠΠ...Π..(3^i*5^j*7^k......all prime numbers where i,j,k,..... runs from 0 to infinity.

III. How primes generate

Prime numbers are odd numbers (except 2). Let's say 1,2,3 only these 3 numbers are given and we are supposed to make natural numbers.

1 2 3 2^2 2*3 but 2^2 and 2*3 both even number. But between two even numbers there must be an odd number . We call it 5 a prime number because no other integer can give birth to this number. Prime number series generates in this way.

Generation of Natural Number Series, given 1,2 and operator multiplication :

After 1,2 there must be a number. Let's call it p_1.

Now we have 1,2,p_1.

Next number must be 2*2 i.e. 2^2

Now we have : 1, 2, p_1, 2^2

Next number is 2*p_1

Now, we have 1, 2, p_1, 2^2, 2*p_1

But between two even numbers there must be an odd. Let's call it p_2.

Now, we have : 1, 2, p_1, 2^2, p_2, $2p_1$

Next number is 2*2^2 or 2^3

So, we have : 1, 2, p_1, 2^2, p_2, $2p_1$, 2^3

But between two consecutive even number there must be an odd number.

The odd number might be p_1*p_2

Now, $2 < p_1 < 2^2$ and $2^2 < p_2 < 2^3$.

Multiplying both the equation : $2^3 < p_1*p_2 < 2^5$

As $p_1*p_2 > 2^3$ then p_1*p_2 cannot fill the gap between $2p_1$ and 2^3.

It might be p_1^2. As $2^2 < p_1^2 < 2^4$

Let's say this number is p_1^2 (Assumption)

Now, we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3

Next number is $2p_2$

Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, $2p_2$

But between two even number there must be an odd.

It might be p_1*p_2

Now, $p_1 > 2 \Rightarrow p_1*p_2 > 2p_2$ (Multiplying both sides by p_2)

And we see that there is least number which can fill the gap between 2^3, $2p_2$

New member must come up. Call it p_3

Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$

Next number is 2*$2p_1$ or 2^2p_1

Now, we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, 2^2p_1

But there must be an odd number between two consecutive even number.

It might be p_1*p_2

Let's say it is p_1*p_2

Now, we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1

Next number is : $2p_1^2$

Now we have, 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, $2p_1^2$

But there must be an odd number between two even number.

It cannot be p_2^2 as $p_2^2 > 2^4$ ($p_2>2^2$) and 2^4 has not yet come.

After p_1*p_2 next odd number can be formed p_2^2 and it cannot fill the gap between 2^2p_1, $2p_1^2$.

So, there must be a new member. Call it p_4.

Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$

Next number is 2*2^3 i.e. 2^4.

Now we have, 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$, 2^4

But there must be an odd number between two even number.

Least odd number can be formed p_2^2 but we have already seen $p_2^2 > 2^4$

So, there must be a new member. Call it p_5.

Now, we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$, p_5, 2^4

Next number is 2*p_3

Manuscript received November 03, 2013.

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Now we have, 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$, p_5, 2^4, $2*p_3$
But between two consecutive even number there must be an odd.
It may be p_2^2 as p_2^2 is the minimum odd number that can be formed after p_1*p_2 and it is also > 2^4.
So, now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$, p_5, 2^4, p_2^2, $2*p_3$
Next number is $2*2p_2$ or 2^2p_2
Now we have, 1, 2, p_1, 2^2, p_2, $2p_1$, p_1^2, 2^3, p_3, $2p_2$, p_1*p_2, 2^2p_1, p_4, $2p_1^2$, p_5, 2^4, p_2^2, $2*p_3$, 2^2p_2
Now, $p_3 > 2p_2 \Rightarrow 2*p_3 > 2^2p_2$ which is contradiction.
So our assumption was wrong.
So, now we have, : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2
Now, we can omit the coloured part.
Next number is : $2*p_2$
Now we have, 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$
Next number is $2*2p_1$ i.e. 2^2p_1
Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, 2^2p_1
But between two consecutive even number there must be an odd number.
Least odd number that can be formed after p_1^2 is p_1*p_2
But $p_1p_2 > 2^2p_1$ (as $p_2>2^2 \Rightarrow p_1p_2>2^2p_1$)
So call for a new member. Let's call it p_4
Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, p_4, 2^2p_1
Next number is : $2*p_3$
Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, p_4, 2^2p_1, $2*p_3$
But between two consecutive even number there must be an odd.
Least number can be formed p_1*p_3
Now, $p_1 > 2 \Rightarrow p_1p_3 > 2*p_3$
So, call for a new member. Let's name it p_5.
Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, p_4, 2^2p_1, p_5, $2*p_3$
Next number is : $2*2^3$ i.e. $(2^2)^2$
Now we have : , 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, p_4, 2^2p_1, p_5, $2*p_3$, $(2^2)^2$
But between two consecutive even number there must be one odd.
The least odd number that can be formed is p_1*p_2 which is less than 2^4
So, it gap must be filled by p_1*p_2
Now we have : 1, 2, p_1, 2^2, p_2, $2p_1$, p_3, 2^3, p_1^2, $2p_2$, p_4, 2^2p_1, p_5, $2*p_3$, p_1*p_2, $(2^2)^2$
Now all pi's are nothing but prime according to the alternative definition of prime given above.
In this way we can form the natural number series.

Observation

1) We can see there are at least one prime between 2^2 and p_1^2.
2) There is at least one prime between p1 and p_1^2 & 2 and 2^2 & 2^2 and 2^3 & 2^3 and 2^4
3) There is at least one prime between $2p_1$ and 2^2p_1.

As this result is born without any assumption we can take it as general case. So we can conclude the following :

Conclusion

1) There is at least one prime between two consecutive square numbers.
2) There is at least one prime between n^p and n^(p+1) where n is prime.
3) The is at lease one prime between m*n^p and m*n^(p+2)

IV. Legendre Conjecture

Between 13^2 (=169) and 14^2 (=196) there are five primes (173, 179, 181, 191, and 193); between 30^2 (=900) and 31^2 (=961) there are eight primes (907, 911, 919, 929, 937, 941, 947, and 953); between 35^2 (=1225) and 36^2 (=1296) there are ten primes (1229, 1231, 1237, 1249, 1259, 1277, 1279, 1283, 1289, and 1291).

The problem is to prove Legendre's Conjecture, which states that there is at least one prime number between every pair of consecutive squares, or find a counter-example.

Solution

From Conclusion 1 we can say Legendre's Conjecture is true.

V. Grimm's Conjecture

Grimm's conjecture states that to each element of a set of consecutive composite numbers one can assign a distinct prime that divides it.

For example, for the range 242 to 250, one can assign distinct primes as follows:

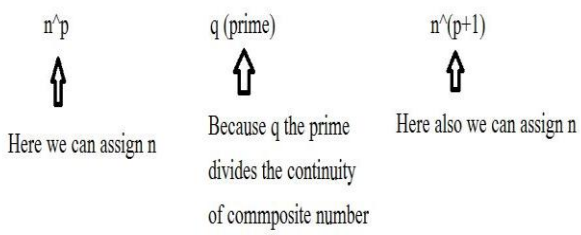
242: 11 **243:** 3 **244:** 61 **245:** 7 **246:** 41 **247:** 13 **248:** 31 **249:** 83 **250:** 5

The problem is to prove the conjecture, or find a counter-example.

Solution

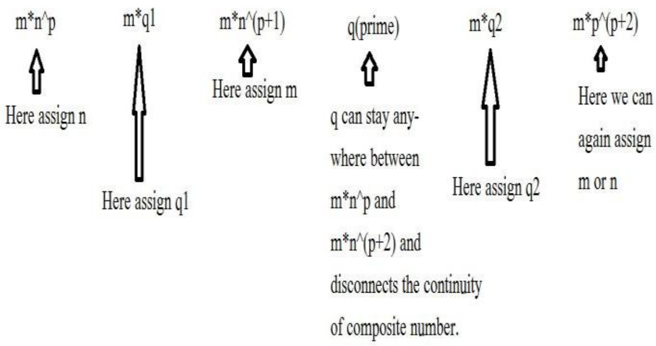
Case 1 :

If we can prove there is at least one prime between n^p and n^(p+1) then we can assign n, to both n^p and n^(p+1) as the image below says :



Case 2

If we can prove there is at least one prime between m*n^P and m*n^(p+2) then we can assign n to m*n^p , m to m*n(p+1) and again n or m to n^(p+2) as the below picture says :



So, from Conclusion 2 & 3 we can say Grimm's Conjecture is true.

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