Performance of FEC codes over AWGN channel for efficient use in Polymer Optical Fiber links

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***Abstract*— In this project, we study the application of Polymer Optical Fiber and Laser sources in High Speed Data Communication. The advantages of using multicarrier modulation schemes like Discrete Multitone (DMT) are studied. We also study the application of Reed Solomon (RS) codes for Forward Error Correction (FEC) in optics. We consider a particular RS code, RS (255,239), of length 255 and having capability to correct upto 8 errors.**

***Index Terms*— Forward Error Correction (FEC), Reed Solomon (RS), Discrete Multitone (DMT), Bose, Chaudhary and Hocquenghem codes (BCH).**

I. Introduction

study in this research could be easily divided into two parts. First is the study of application of Forward Error Correction in general and Reed Solomon in particular in the field of optics. Second is the study of POF fibers with regards to their applications in short range data communication. We also study the need of using multicarrier modulation schemes like Discrete Multitone (DMT) therein.

Let's first talk about the need of Forward Error Correction in optics. There has been tremendous increase in the data rates along the optical fiber thanks to the advances in optical devices and enabling technologies. Technologies like DWDM (Dense Wavelength Division Multiplexing) have helped in high speed communications which in turn has made various luxuries like video conferencing, live video streaming etc. possible. But as we move on to high data rates, the performance of the channel degrades considerably. Several impairments like PMD (Polarisation Mode Dispersion), CD (Chromatic Dispersion), and various other fiber non-linearities come into picture. These impairments cause the data to get corrupted and it is here where FEC (Forward Error Correction mechanisms) come to help.

Now let us consider Polymer Optical Fibers. POFs have enjoyed wide applications in the areas of automobiles and medicine for quite some time now, but its application in the field of FTTH (Fiber to the Home) is quite new. Though POFs have very high attenuation as compared to glass fibers, their application in short range data communication is a viable solution because of their easy-to-use setup facilitated by high core diameter. It was way back in 1992, when Bates of IBM demonstrated transmission over 100m SI-POF at 500 Mb/s [8]. It was no looking back since then. There has been constant innovation in the field of POFs and the following table gives some parameters of various POFs:

Strategy	standard POF	large bandwidth PMMA-POF	perfluorinated GI-POF
available fibers	3 Japanese companies	Mitsubishi: MSI-POF Optimedia: GI-POF Asahi Chemical: MC-POF	Asahi Glass Company Nexans France Chromis Fiberoptics
typ. parameter	$Ø_{core}$: 1000 µm NA: 0.48 ± 0.02 bandwidth: 40 MHz·100m 130 dB/km at 650 nm 90 dB/km at 520 nm	$Ø_{core}$: 750 µm NA: 0.40 ± 0.02 bandwidth: 500 MHz·100m 160 dB/km at 650 nm	$Ø_{core}$: 120 µm NA: 0.22 bandwidth: 5000 MHz·100m 30 dB/km at 850 nm 20 dB/km at 1300 nm
potential (typical)	100 Mbit/s over 100 m 1000 Mbit/s over 50 m	1000 Mbit/s over 100 m	10,000 Mbit/s over 100 m 1,000 Mbit/s over 500 m
price	approx. 10 ct./m	potentially like standard POF	?

Table1.1: Characteristics of different POF.

High data rates and spectral efficiency makes us look for advanced communication technologies.

	PLASTIC	GLASS	COPPER
Connection	Easy to connect	Takes longer, require training	HIGH
Handling	Easy	Require training	Easy
Flexibility	Flexible	Brittle	Flexible
Component Costs	Potentially Low	More Expensive	Low
Loss	High-medium	Medium-low	High
Wavelength operating range	Visible	Infrared	NA
Bandwidth	High(0.4)	Low(0.1-0.2)	NA
System costs	Low overall	High	Medium

Table 2: Comparison of POF with Glass fiber and Copper wires over various parameters

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One specific technology called Discrete Multitone (DMT) modulation has proved quite promising. This technology combined with advanced modulation formats like M-ary Quadrature Amplitude Modulation (QAM) is a hot topic in the current research scenario and high data rates have been achieved using them.

II. Proposed Work

Discrete Multitone Modulation (DMT) is a kind of multicarrier modulation scheme which transmits a high data rate serial signal over a number of slow parallel subcarriers; essentially maintaining the data rate.

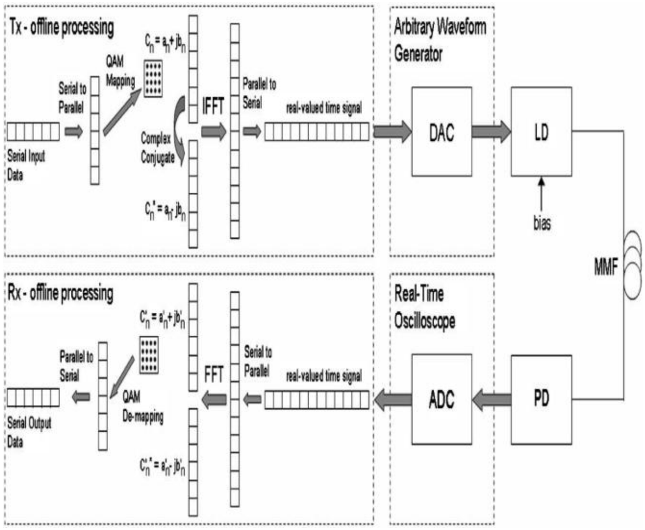

Figure 1: Schematic of DMT modulation and experimental generate colored noise.

The serial data input is divided into N parallel subcarriers which are further mapped to M-QAM constellations. The C_n's in the figure are the complex values of M-QAM. The modulator of DMT is implemented using Inverse Fats Fourier Transform (IFFT) while the demodulator is implemented using Fast Fourier Transform (FFT).

On taking IFFT of N symbols a complex valued sequence is obtained, which is the case in OFDM. The output of IFFT of N symbols is as follows:

$$Sk = \frac{1 \sum_0^{N-1} Cn e^{\frac{j2\pi kn}{N}}}{\sqrt{N}} \quad \text{.........K=0.....N-1}$$

To have a real valued signal, we need to take 2N point IFFT where the input values will satisfy the following conditions:

$$C_{2N-n} = C_n^{*} \text{.......n=1.......N-1}$$

$$Im\{C_0\}=Im\{C_n\}=0$$

These parallel data streams are converted to serial data stream before transmission. The C_n are obtained by demodulation using the FFT algorithm.

$$C_n = \frac{1 \sum_0^{2N-1} Ske^{-\frac{j2\pi kn}{2N}}}{\sqrt{2N}} \quad \text{......n=0.....2N-1}$$

Adaptive Constellation Mapping of Cn

After allocation of SNR over the subcarriers based on the frequency response of the system, bits are allocated over the subcarriers. The following figure shows the number of bits allocated or the order of QAM used for a given range of frequencies or equivalently subcarriers in a typical 512 subcarrier system

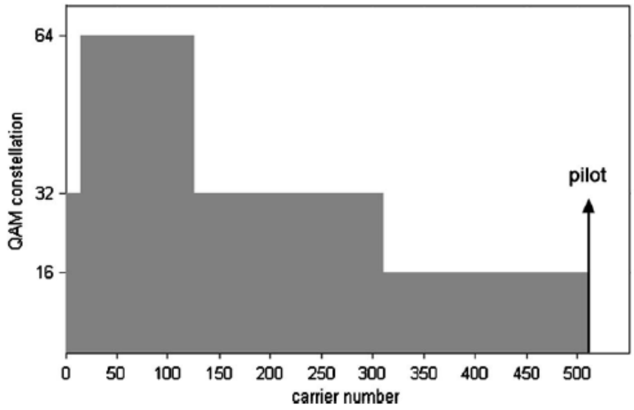

Figure 2: QAM constellation vs. Carrier number

The use of Cyclic Prefix- After parallel to serial conversion of a DMT frame, a portion of the last part of the frame is added in the front as shown in the figure below

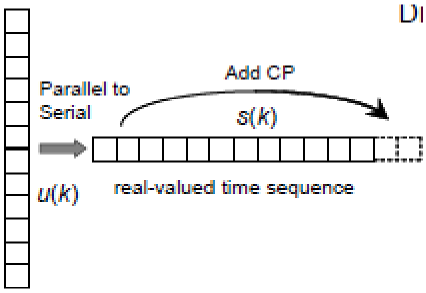

Figure3: Cyclic Prefix

This repeated portion is called as a cyclic prefix. This cyclic prefix is useful in combating modal dispersion. The length of the cyclic prefix is chosen such that it is greater than the largest delay spread. Thus, dispersion will not be able to affect the actual useful DMT frame.Review Stage

Synchronization- For the receiver to be able to distinguish between different DMT frames and to avoid faulty demodulation of the transmitted sequence, synchronization is a must. It can obtain by the following ways:

A. *By using the cyclic prefix of every DMT frame*
B. *By sending preambles*

The cyclic prefix of every DMT frame is correlated with its time shifted version. If the CP correlates with itself, a large value of correlation will be achieved .As shown in figure below, parts a and b are identical and are a part of preamble sequence.

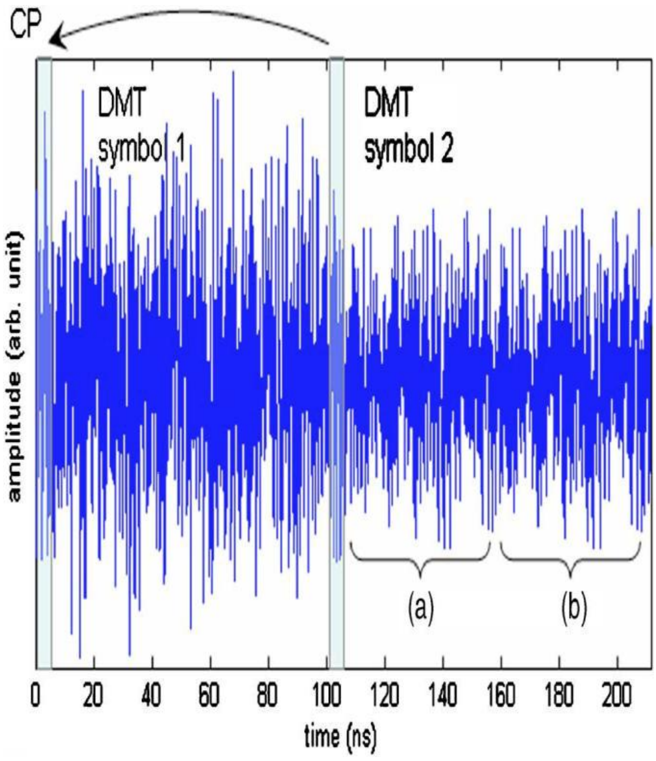

Figure 4: Transmitted DMT frame consisting of a preamble and information frame

The importance of Peak to Average Power Ratio (PAPR)

In Discrete Multitone Modulation, sometimes high values of peak powers are generated due to constructive interference of the subcarriers. For a DMT frame the PAPR could be given by the following formula

$$\mathrm{PAPR}_{\mathrm{dB}} = 10 \times \log_{10}\left(\frac{\max\limits_{0\le t\le T}|s(t)|^2}{E[|s(t)|^2]}\right)$$

Here the E[] operator denotes the average. When N subcarriers add up constructively, the peak power would be N times the average power.

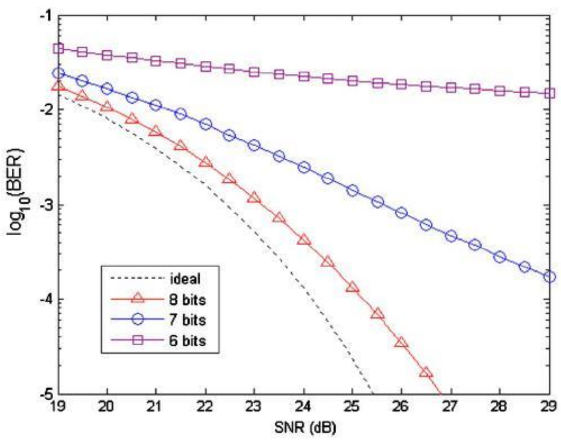

Figure 5 : Simulated BER vs. SNR per DMT frame for different AD- and DA- Converter resolutions

A large PAPR is disadvantageous in the sense that the DAC's and the ADC's used in the system have limited precision. Thus they might not be able to accommodate such high values of PAPR. For e.g. take a system with 511 subcarriers. Thus the maximum value of PAPR for DMT frames would be $10\log_{10}(511) = 27$ dB.The figure avobe shows the precision of DAC and ADC required to obtain appropriate BER values.

We see from the figure that the probability that PAPR values will cross even 15 dB is less than 10^{-4}. Thus it is not necessary for the DAC and the ADC to accommodate dynamic range for maximum value of PAPR.

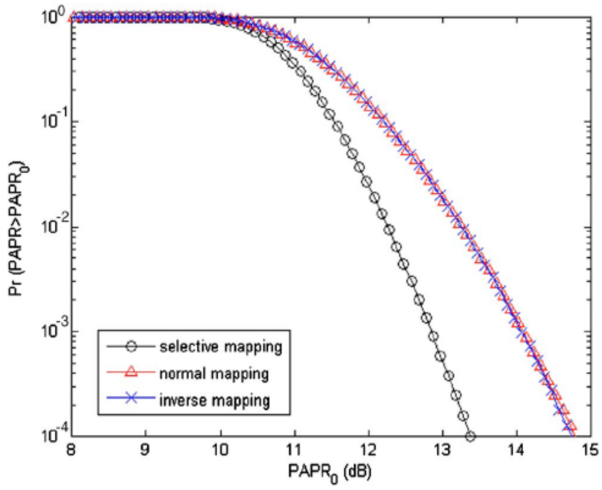

Figure 6.: Simulated CCDF of PAPR in DMT transmission with and without selective mapping

Clipping of DMT signal

We can achieve optimum performance by limiting the dynamic range of DAC and ADC to a proper value. To limit the DMT signal to the dynamic range of DAC and ADC, clipping is employed. A simple clipping could be easily implemented by the following algorithm

$$s_{\mathrm{clip}}(t) = \begin{cases} s(t), & |s(t)| \le A \\ A\exp(j\arg\{s(t)\}), & |s(t)| > A' \end{cases}$$

Where A is the level at which the DMT signal is clipped. .

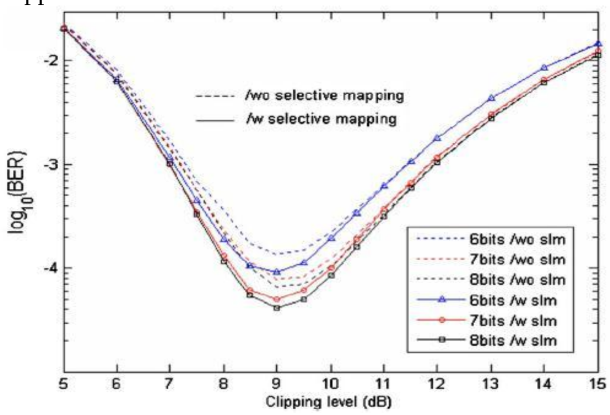

Figure 7: Simulated BER at constant receiver noise power

versus clipping level for different AD- and DA- converter resolutions

The following figure shows the performance of the system with respect to the clipping levels employed for different resolutions of DAC and ADC.

Selective Mapping- This technique is cost efficient as compared to those wherein the PAPR reduction without introducing distortion reduces PAPR to such an extent that clipping is unnecessary.
This is obtained by employing the following symmetry conditions on some second Input say Dn:

$D_{2N-n} = -D_n^*, n=1 \ldots\ldots\ldots N-1$

$\mathrm{Im}\{D_0\} = \mathrm{Im}\{D_n\} = 0$

Combining the two orthogonally and performing IFFT on them we get the following

$$s_k = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} X_n \exp\left(j2\pi k \frac{n}{2N}\right),$$

$$s_k = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} (C_n + jD_n) \exp\left(j2\pi k \frac{n}{2N}\right),$$

$$s_k = \frac{1}{\sqrt{2N}} \left[\sum_{n=0}^{2N-1} C_n \exp\left(j2\pi k \frac{n}{2N}\right) + j \sum_{n=0}^{2N-1} D_n \exp\left(j2\pi k \frac{n}{2N}\right) \right],$$

$$s_k = s_k^{C_n} + j s_k^{D_n},$$

Now we can implement two different mappings on these two inputs.

Reed Solomon codes- Forward Error Correction could be defined as a method to detect and/or correct errors in data transmission over a faulty channel. This is usually done by adding redundant data to the information carrying data. This technology is very much developed and has wide applications in the wireless world. In Optics, it was first used in WDM (Wavelength Division Multiplexing) to combat ASE (Amplified Spontaneous Emission) a form of noise associated with optical amplifiers. Today, a wide variety of FEC systems are available for error correction in the optical transmission networks. They differ in features like the amount of redundancy, the coding gain achieved, the BER performance etc. In this report we will discuss a very useful FEC mechanism called the RS (Reed Solomon) codes.

Encoding RS Codes- Suppose α is a primitive element in GF(q), i.e. $\alpha^{q-1} = 1$. Now, $\alpha, \alpha^2, \alpha^3, \ldots, \alpha^{2t}$ are all **the** roots of the generator polynomial of the t error correcting RS code. The minimal polynomial $\phi_i(X)$ of α^i would simply be X- α^i since α^i is an element of GF(q). Thus the generator polynomial g(X):

$$g(X) = (X-\alpha)(X-\alpha^2)(X-\alpha^3) \ldots (X-\alpha^{2t})$$

$$= g_0 + g_1X + g_2X^2 + \ldots + g_{2t-1}X^{2t-1} + X^{2t}$$

The t-error correcting RS code with symbols from *GF(q)* :
Block Length: $n = q-1$
Number of Parity Check Symbols: $n-k = 2t$
Dimension: $k = q-1-2t$
Minimum Distance: $d_{min} = 2t+1$

Decoding RS Codes-For decoding RS code, we not only need to calculate error locations but also need to calculate the error values. Let the transmitted code be
$v(X) = v_0 + v_1X + \ldots + v_{n-1}X^{n-1}$
Let the received code be
$r(X) = r_0 + r_1X + \ldots + r_{n-1}X^{n-1}$
The error polynomial can now be written as
$e(X) = v(X)-r(X) = e_0 + e_1X + \ldots + e_{n-1}X^{n-1}$
Suppose the errors are located at locations $X^{j1}, X^{j2}, \ldots, X^{jv}$ and have values $e_{j1}, e_{j2}, \ldots, e_{jv}$.
The error polynomial can then be written as
$e(X) = e_{j1}X^{j1} + e_{j2}X^{j2} + \ldots + e_{jv}X^{jv}$.

The outline for RS decoding could be given as follows:

1. First we compute the syndrome $(S, S_2, S_3 \ldots S_{2t})$
2. Next we determine the error location polynomial $\sigma(X)$
3. Then we determine the error value evaluator
4. Having obtained error locations and error values, we could perform error correction

The first two steps mentioned above are to done in a similar fashion as done in the case of BCH codes. So now we will see how to determine the error value evaluator. Let us define the syndrome polynomial *S(X)* as follows

$$S(X) = \sum_{j=1}^{\infty} S_j X^{j-1}$$

But we know the co-efficient of only the first 2t terms. Hence for $1 \le j < \infty$, we define a term

$$S_j = \sum_{l=1}^{v} \delta_l \beta_l^j$$

Where δ and β are error values and error locations respectively

$$\mathbb{S}(X) = \sum_{j=1}^{\infty} X^{j-1} \sum_{l=1}^{\nu} \delta_l \beta_l^j$$
$$= \sum_{l=1}^{\nu} \delta_l \beta_l \sum_{j=1}^{\infty} (\beta_l X)^{j-1}.$$

$$\mathbb{S}(X) = \sum_{l=1}^{\nu} \frac{\delta_l \beta_l}{1 - \beta_l X}.$$

Now, let us look at the product $\sigma(X).S(X)$

$$\sigma(X)\mathbb{S}(X) = (1 + \sigma_1 X + \cdots + \sigma_\nu X^\nu) \cdot (S_1 + S_2 X + S_3 X^2 + \cdots)$$
$$= S_1 + (S_2 + \sigma_1 S_1)X + (S_3 + \sigma_1 S_2 + \sigma_2 S_1)X^2 + \cdots +$$
$$(S_{2t} + \sigma_1 S_{2t-1} + \cdots + \sigma_\nu S_{2t-\nu})X^{2t-1} + \cdots$$

$$\sigma(X)\mathbb{S}(X) = \left\{\prod_{i=1}^{\nu}(1-\beta_i X)\right\}\cdot\left\{\sum_{l=1}^{\nu}\frac{\delta_l\beta_l}{1-\beta_l X}\right\}$$
$$= \sum_{l=1}^{\nu}\frac{\delta_l\beta_l}{1-\beta_l X}\cdot\prod_{i=1}^{\nu}(1-\beta_i X)$$
$$= \sum_{l=1}^{\nu}\delta_l\beta_l \prod_{i=1,i\neq l}^{\nu}(1-\beta_i X).$$

Now we define another parameter as $Z_0(X)$:

$$\mathbb{Z}_0(X) \triangleq \sum_{l=1}^{\nu}\delta_l\beta_l \prod_{i=1,i\neq l}^{\nu}(1-\beta_i X)$$

We can find the error value at location β_k by the following:

$$\delta_k = \frac{-\mathbb{Z}_0(\beta_k^{-1})}{\sigma'(\beta_k^{-1})}.$$

Reed-Solomon Codes in Optical Communication

Reed Solomon codes have traditionally been used in CD's and satellite communications. Today, Reed Solomon codes are being concatenated with other convolution codes to improve BER and also attain a low level of complexity .The history of the development of Forward Error Correction and Optical Communication could be easily captured in the figure below:

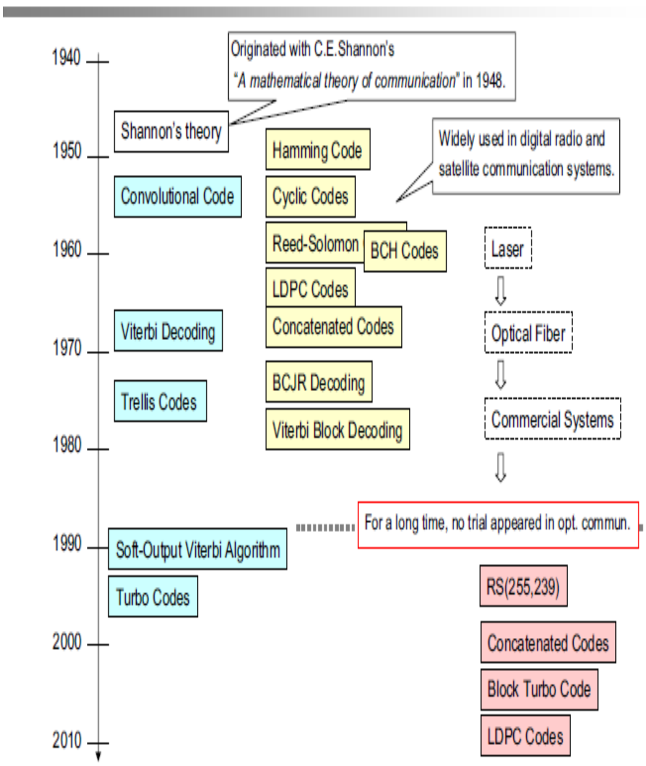

Figure 8: Development of FEC in Optical Communication across time

III. RESULTS

Performance of Reed Solomon code is evaluated for awgn channel across various parameters

A. Performance of Reed Solomon code as the order of the code varies but error correcting capability remains same:

We implemented the code keeping the error correcting capability same, 8 in each case, but varying the order of code. We varied the order from m = 6 to m = 9 thus implementing RS (63,47), RS (127, 111), RS (255, 239) and RS (511, 495).

a.Performance of RS (63, 47) code: RS (63, 47) is a code of order 6 and has an error correcting capability of 8. The following plots were obtained after its implementation.

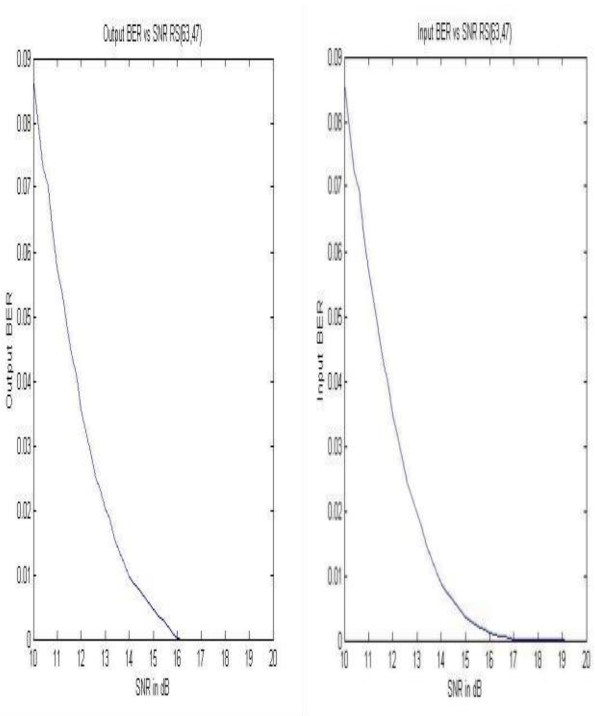

Figure 9.1: Performance of RS (63, 47)

b. Performance of RS (127, 111) code: RS (127, 111) is a code of order 7 and has an error correcting capability of 8. The following plots were obtained after its implementation.

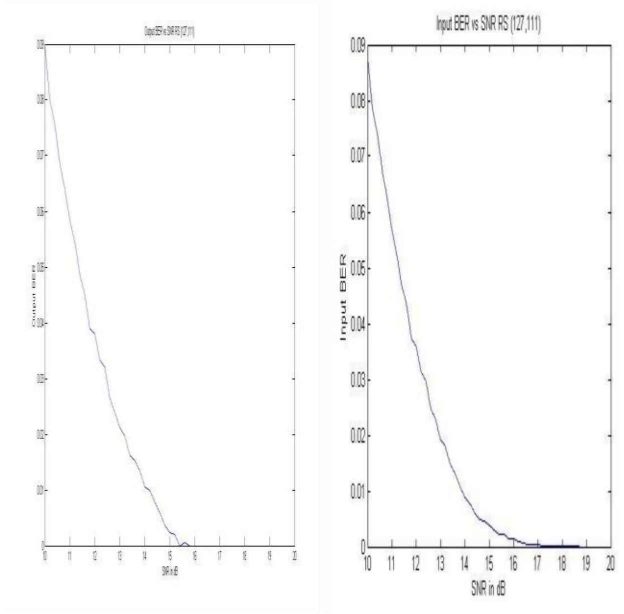

Figure 9.2: Performance of RS (127, 111) code

c. Performance of RS (255,239) code: RS (255, 2 5 9) is a code of order 8 and has an error correcting capability of 8. The following plots were obtained after its implementation.

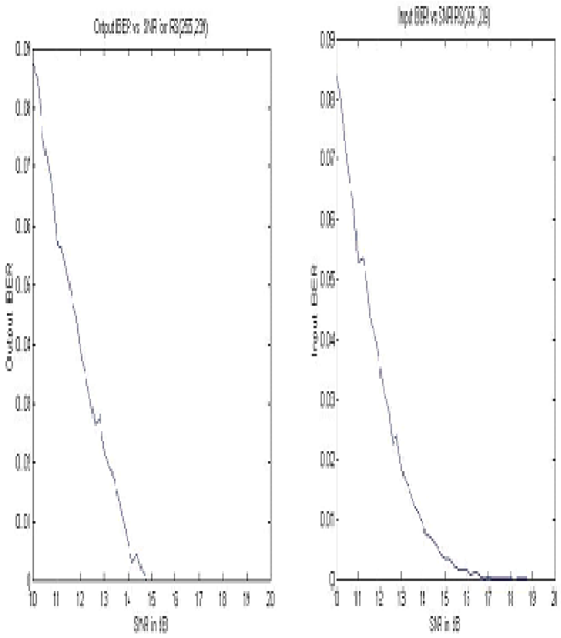

Figure 9.3: Performance of RS (255, 239) code

d. Performance of RS (511, 495) code: RS (255, 239) is a code of order 8 and has an error correcting capability of 8. The following plots were obtained after its implementation.

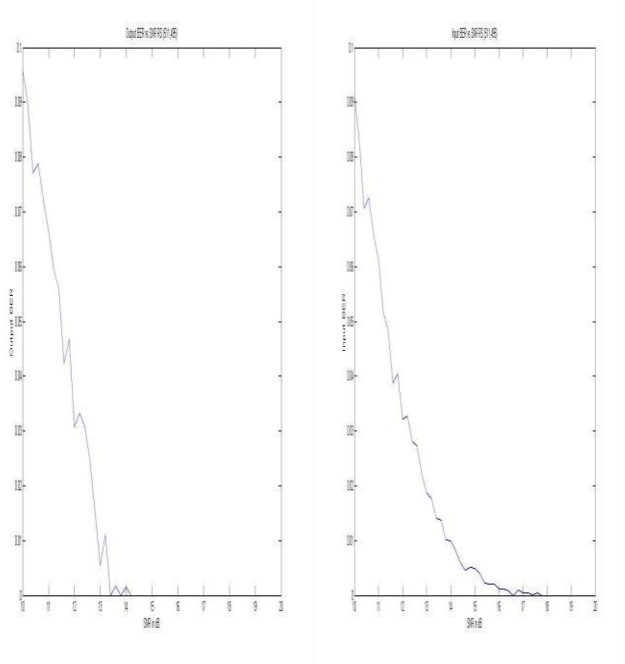

Figure 9.4: Performance of RS (511, 495) code

The Input BER goes to zero at an SNR of about 17dB in each case. The SNR where Output BER goes to zero is called as 'SNR Threshold'. The above performance could be tabulated as below:

R	SNR Threshold (dB)	Coding Gain (dB)
RS(63, 47)	16	1
RS(127, 111)	15.8	1.2
RS(255, 239)	14.8	2.2
RS(511, 495)	14.2	2.8

Table 6.5: Performance parameters of various codes with same error correcting capability

B. Performance of Reed Solomon code as the redundancy of the code varies but order remains same:

We implemented the code keeping the order same, 8 in each case, but varying the order of code. We varied the redundancy from k = 5 to k = 8 thus implementing RS (255, 245), RS (255,243), RS (255, 241) and RS (255, 239).

a. Performance of RS (255, 245) code:

RS (255, 245) is a code of order 8 and has an error correcting capability of 5. The following plots were obtained after its implementation.

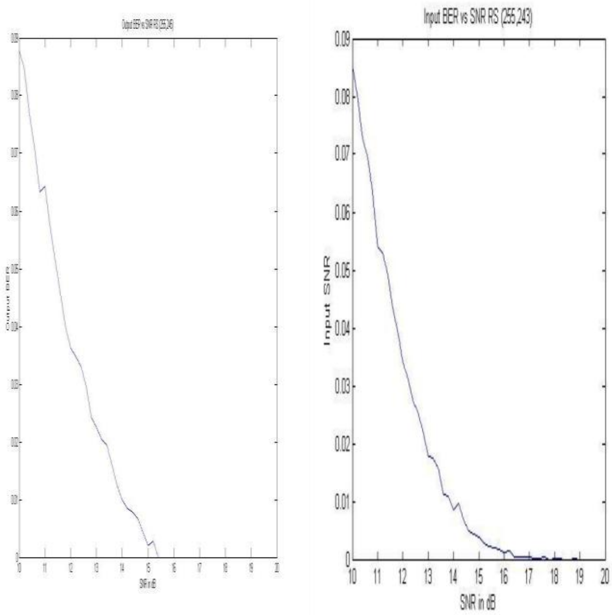

Figure 9.6: Performance of RS (255, 245)

code b. Performance of RS (255, 243) code:

RS (255, 245) is a code of order 8 and has an error correcting capability of 6. The following plots were obtained after its implementation.

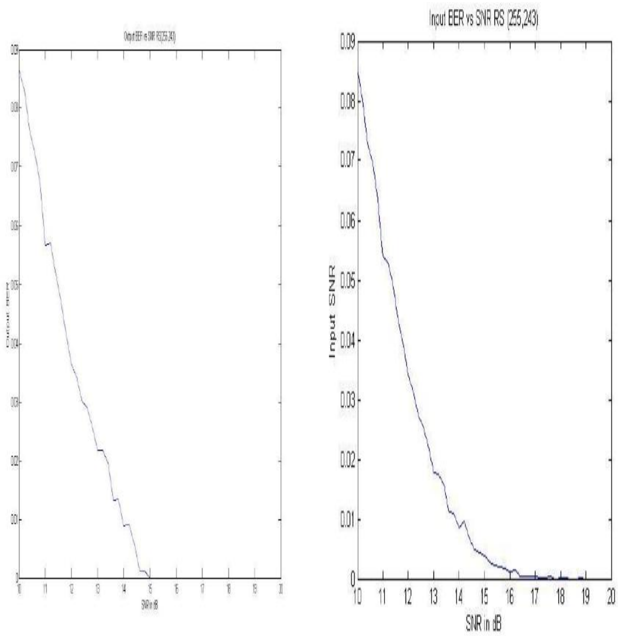

Figure 9.7: Performance of RS (255, 243) code

c. Performance of RS (255, 241) code:

RS (255, 241) is a code of order 8 and has an error correcting capability of 7. The following plots were obtained after its implementation.

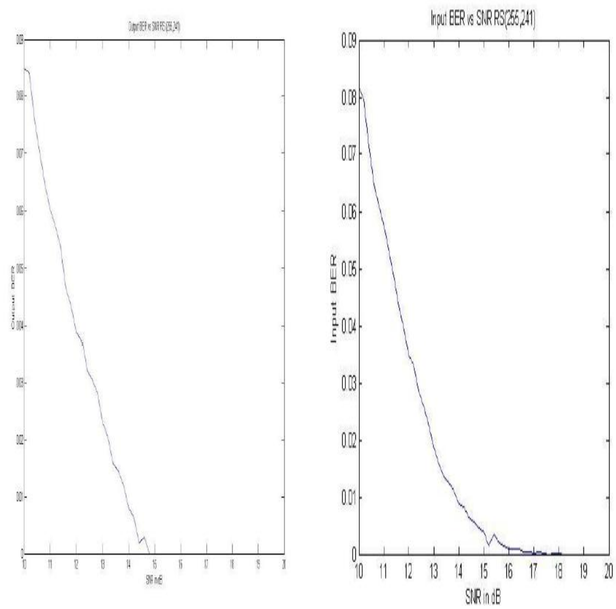

Figure 9.8: Performance of RS (255, 243)

d. Performance of RS (255, 239) code:

RS (255, 239) is a code of order 8 and has an error correcting capability of 8. The following plots were obtained after its implementation.

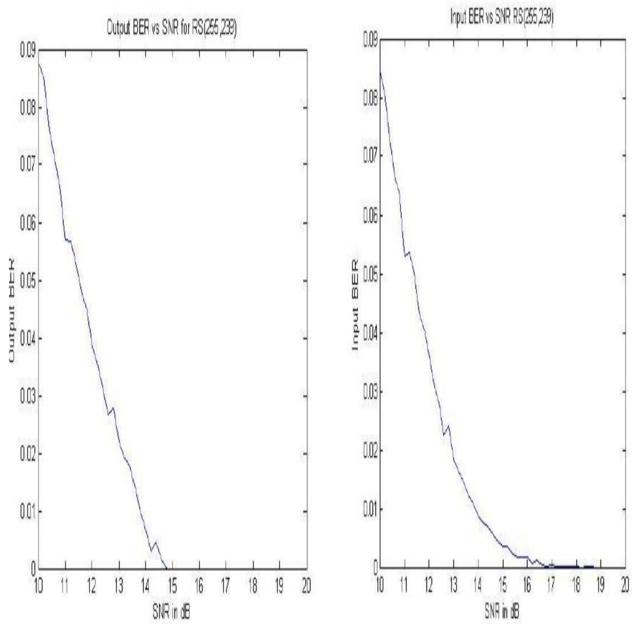

Figure 9.9: Performance of RS (255, 239) code

RS Code	SNR Threshold (dB)	Coding Gain (dB)
RS(255, 245)	15.4	1.6
RS(255, 243)	15	2
RS(255, 241)	14.8	2.2

RS(255, 239)	14.8	2.2

Table 6.2: Performance parameters of various codes

IV. Conclusion

The above results show that Reed Solomon codes are very flexible and are able to achieve significant BER improvement. We see that, for the same error correcting capability, as the order of the code increases the SNR at which Output BER becomes zero decreases. Thus, the coding gain achieved increases.

We are also able to prove that for the same order as the redundancy of the code increases, we are able to achieve improved BER performance. We are able to achieve higher coding gain by increasing the redundancy. But in the process we are also reducing the code rate.

We also see that, even if we are able to achieve the same error correcting capability for lower order of Reed Solomon codes and hence low complexity in the circuitry, the code rate is significantly low for them. And for the same error correcting capability if we move to higher order of Reed Solomon codes, though we gain in terms of code rate but we also lose in terms of complexity of the circuitry. This classical trade-off is somewhat resolved at RS (255, 239) which explains for its popularity for Forward Error Correction in Optical systems.

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