Design Template Creation for Minimum Oil Film Thickness for Crankshaft Bearing In a Single Cylinder Four Stroke Diesel Engine Using Mathcad

Mr. Snehal B. Bhadke, Mr. Amit U. Chaudhari, Dr. S. B. Sharma

Abstract — Hydrodynamic journal bearings are critical power transmission components that are carrying increasingly high loads because of the increasing power density in various machines. Therefore, knowing the true operating conditions of hydrodynamic journal bearings is essential for machine design. Oil film thickness is one of the key operating parameters describing the operating conditions in hydrodynamic journal bearings. Measuring the oil film thickness in bearings has been a demanding task and therefore the subject has been studied mainly by mathematical means.

The aim of this study was to determine the oil film thickness in real hydrodynamic journal bearings under realistic operating conditions. The study focused on engine bearings. Calculations were carried out to determine the oil film thickness and to understand its relationship with other operating parameters. Analytical calculations are highly complex and very difficult to perform using MS Excel. So Mathcad is a good solution for performing this type of complex calculations and formulations.

Once the template is ready, it can be used for other engine bearings to determine the minimum oil film thickness by analytical means. After performing whole calculations, the template is used to integrate with actual CAD model of the components. The results can be used in the development and validation of mathematical methods for research into hydrodynamic journal bearings. Integration process for ball bearing is shown in detail using Mathcad and Creo. Then actual CAD model will be automatically modified as dimension in Mathcad worksheet modifies after successful regeneration in Creo parametric. Design template calculations results are in good agreement with results given by “KISSsoft” software.

Index Terms — Hydrodynamic lubrication, Integration procedure between Mathcad and Creo, Journal bearing, Mathcad formulation, Oil film pressure, Oil film thickness.

I. INTRODUCTION

The power density in various machines, for example in internal combustion engines, is increasing year by year due to growing demands for mechanical and economic efficiency. In machine design, one of the consequences of an increase in power density is that critical power transmission components have to carry increasingly high loads.

Hydrodynamic journal bearings are typical critical power transmission components that carry high loads in different machines. In machine design, therefore, it is essential to know the true or expected operating conditions of the bearings. These operating conditions can be studied by mathematical means, for example in field or laboratory tests with engines and by calculation.

Numerous studies of the operating conditions of hydrodynamic journal bearings have been made during the last decades. Still, the case is far from closed. For example, there are a limited number of studies that carry out an in-depth examination of the true operating conditions of bearings in true-scale experiments. There is also a need for experimental studies to verify the theoretical ones.

The operating conditions of hydrodynamic journal bearings can be described by a set of tribological variables called key operating parameters. The key operating parameters most directly related to the bearing lubricant–shaft contact are the oil film temperature, oil film thickness and oil film pressure. These three key parameters can be determined by mathematical means with varying levels of complexity. Until now, oil film pressure in hydrodynamic journal bearings has been studied mainly by mathematical means, because the experimental determination of oil film pressure has been a demanding or even an unfeasible task. Under real operating conditions, there are typically many practicalities that complicate the experimental determination of true oil film pressure in a certain point or at a certain moment. The oil film may be extremely thin and therefore sensitive to different disturbing factors, for example defects in geometry. In addition, the level of the oil film pressure may be extremely high or have a high level of dynamic variability.

II. BASICS OF THE OPERATION OF HYDRODYNAMIC JOURNAL BEARINGS

Lubrication reduces friction between two surfaces (such as sliding surfaces of a bearing and a shaft) in relative motion. It is typically categorised as boundary, mixed and hydrodynamic lubrication, as shown in example by Heywood (1988), Becker (2004) [12] and Gleghorn and Bonassar (2008) [11]. When a journal bearing operates under boundary lubrication, the sliding surfaces of the bearing and shaft are practically in direct contact and friction is at its highest level. Lower friction levels are achieved through the use of mixed lubrication, where the sliding surfaces are partially separated by the lubricant, and of hydrodynamic lubrication, where the sliding surfaces are completely separated by the lubricant.
To illustrate how friction varies under different lubrication conditions, Stribeck curves have been used widely in different engineering sciences. In Stribeck curves, friction coefficient is presented as a function of a dimensionless parameter calculated from dynamic viscosity, angular speed and pressure (Figure 1). The above-mentioned parameter is typically called the duty parameter or Hersey number. The minimum of the friction coefficient is reached at the critical value of the duty parameter, at the dividing line between the mixed and hydrodynamic lubrication zones. Heywood (1988) [12] presented a Stribeck curve for a journal bearing. Methods for the calculation of Stribeck curves were studied by de Kraker et al. (2007) [16]. They calculated the friction coefficient as a function of the journal frequency at different values of the projected bearing pressure.

The bearing unit consists of the bearing, housing, shaft and supporting bearings. The housing in which the bearing is placed for testing typically has a simplified cubic or cylindrical design, but housings of real types have also been used. A large high precision roller bearing with both radial and axial load carrying capacity is a common supporting bearing type used in low and medium speed applications. These have been used, for example, by Savaşkan et al. (2002) [3]. In high speed or extreme load applications, sliding bearings have been used as an alternative, for example by Okamoto et al. (2000) [3] and Zhou et al. (2004) [10].

A hydrodynamic journal bearing (Figure 2) is designed to operate normally under hydrodynamic lubrication, where hydrodynamic pressure (Figure 3) in the lubricant keeps the sliding surfaces of the bearing and shaft separated from each other. The hydrodynamic pressure is caused by the sliding motion.

The calculation was made in three phases. In the first phase, the Sommerfeld number $S_n$ (a dimensionless parameter used in bearing performance calculations) was determined approximately by the following equation, based on the measurement data:

$$S_n = \frac{P}{r^2 n \mu}$$

Where, $P$ = Load on the bearing in N, $r$ = Radius of the bearing in m, $n$ = Angular velocity in rad/s, $\mu$ = Dynamic viscosity in kg/m/s.

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\[ S_0 = FC^2 / DW\eta \omega \quad \ldots \quad (2) \]

Where,  
- \( F \) = Radial Bearing Load.  
- \( C \) = Relative Bearing Clearance.  
- \( D \) = Diameter of the Bearing.  
- \( W \) = Width of the Bearing.  
- \( \eta = \mu \rho = \) Dynamic Viscosity.  
- \( \mu = \) Kinematic Viscosity.  
- \( \rho = \) Density of Oil.  
- \( \omega = \) Hydrodynamic Angular Velocity.

The density \( \rho \) in equation (1) was calculated as a function of the temperature. In the second phase, the relative eccentricity \( \varepsilon \) was determined approximately as a function of the Sommerfeld number \( S_0 \) and the width-to-diameter ratio \( B/D \) of the bearing (see Appendix A). The approximation was made for a plain bearing with the width-to-diameter ratio \( B/D = 32 \text{ mm} / 85 \text{ mm} = 0.376 \), and the relative eccentricity \( \varepsilon \) was calculated by the following approximate equation for \( 1 \leq S_0 \leq 200 \):

\[ \varepsilon = k_1S_0^{k_2} \quad \ldots \quad (3) \]

Where,  
- \( k_1 \) = Coefficient.  
- \( S_0 \) = Sommerfeld Number.  
- \( k_2 \) = Coefficient.  

The values of the coefficients \( k_1 \) and \( k_2 \) in the equation (3) are presented in Table 1.

**Table 1. Values of the coefficients \( k_1 \) and \( k_2 \) with different values of the Sommerfeld number \( S_0 \).**

<table>
<thead>
<tr>
<th>( S_0 )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \leq S_0 &lt; 10 )</td>
<td>0.798</td>
<td>0.073</td>
</tr>
<tr>
<td>( 10 \leq S_0 \leq 100 )</td>
<td>0.897</td>
<td>0.022</td>
</tr>
<tr>
<td>( 100 &lt; S_0 \leq 200 )</td>
<td>0.980</td>
<td>0.0028</td>
</tr>
</tbody>
</table>

In the third phase, the following equation was used to calculate the minimum oil film thickness \( h_0 \) as a function of the bearing diameter, relative bearing clearance and relative eccentricity:

\[ h_0 = \left[ \frac{1}{2} \right] DC (1 - \varepsilon) \quad \ldots \quad (4) \]

Where,  
- \( D \) = Diameter of the Bearing.  
- \( C \) = Relative Bearing Clearance.  
- \( \varepsilon \) = Relative Eccentricity.  

Due to the use of a simple calculation method, it can be estimated that the relative error in the calculated minimum oil thickness was high, about \( \pm 10\% \), when the results are compared to values determined by detailed calculation methods.

The oil used for measurement of minimum oil film thickness is SAE 15W-40 with following lubricant properties:

**Table 2. Properties of SAE 15W-40 oil.**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Reference</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity grade</td>
<td>SAE J 300</td>
<td>15W-40</td>
</tr>
<tr>
<td>Density at 20 °C</td>
<td>ASTM D 1298</td>
<td>875 kg/m³</td>
</tr>
<tr>
<td>Viscosity at 100 °C</td>
<td>ASTM D 445</td>
<td>14.5 mm²/s</td>
</tr>
<tr>
<td>Viscosity at 40 °C</td>
<td>ASTM D 445</td>
<td>110 mm²/s</td>
</tr>
<tr>
<td>Pour point</td>
<td>ASTM D 97</td>
<td>-27 °C / -17 °F</td>
</tr>
<tr>
<td>Flash point</td>
<td>ASTM D 92</td>
<td>224 °C / 435 °F</td>
</tr>
</tbody>
</table>

The procedure for determination of minimum oil film thickness by using PTC Mathcad Prime software is as follows:

![Figure 4. Determination of minimum oil film thickness using PTC Mathcad Prime software.](image-url)
IV. DESIGN METHODOLOGY FOR MATHCAD

In this section we will discuss the design methodology used to create the procedure for the diesel engine components using Mathcad software. The design process for determination of minimum oil film thickness for plain journal bearing consists of following steps:

Consider,

\[ F = \text{Radial load on bearing in N.} \]
\[ c = \text{Clearance in bearing in mm.} \]
\[ D = \text{Diameter of bearing in mm.} \]
\[ d = \text{Diameter of journal in mm.} \]
\[ C = \text{Relative clearance in bearing.} \]
\[ W = \text{Width of bearing in mm.} \]
\[ T_{\text{REF}} = \text{Reference temperature in K.} \]
\[ \rho_{\text{REF}} = \text{Density of oil at reference temperature in kg/mm}^3. \]
\[ T_{\text{IN}} = \text{Oil inlet temperature in K.} \]
\[ T_{\text{OUT}} = \text{Oil outlet temperature in K.} \]
\[ T = \text{Service temperature in K.} \]
\[ \rho_T = \text{Density of oil at service temperature in kg/mm}^3. \]
\[ \mu = \text{Kinematic viscosity in mm}^2/\text{s.} \]
\[ \eta = \text{Dynamic viscosity in Pa\cdot s.} \]
\[ \omega = \text{Speed of revolution in rpm.} \]
\[ S_0 = \text{Sommerfeld number.} \]
\[ \varepsilon = \text{Relative eccentricity.} \]
\[ h_0 = \text{Minimum oil film thickness in \( \mu m \).} \]
\[ K_1, K_2 = \text{Constants as per the range of Sommerfeld number.} \]

Diameter of journal is given by

\[
d = D - c
\]

Relative clearance in bearing is given by

\[
C = \frac{c}{d}
\]

Service temperature is given by

\[
T = \frac{(T_{\text{IN}} + T_{\text{OUT}})}{2}
\]

Density of oil at service temperature is given by

\[
\rho_T = \rho_{\text{REF}} \left[1 - 0.0007 \left(T_{\text{REF}} - T_{\text{REF}}\right)/K\right]
\]

Constants for 15W-40 oil at standard working condition [17] are given by

\[
B = \left[\log(\mu) \log(\mu/\left(mm^2/s\right)+0.7)\right] - \log(\log(\mu/(mm^2/s)+0.7))/\log(T_{\text{REF}}) - \log(T_{\text{REF}})
\]

\[
A = \left[\log(\mu) \log(1/(mm^2/s)+0.7)\right] + B \left[\log(T_{\text{REF}}) / K\right]
\]

Kinematic viscosity of oil is given by

\[
\mu = 10^B \left[10^{A \log(\mu/\left(mm^2/s\right)+0.7)}\right] - 0.7 \ mm^2 / s
\]

Dynamic viscosity of oil is given by -

\[
\eta = \mu \rho_T
\]

Sommerfeld number is given by

\[
S_0 = \frac{FC^2}{DW\eta\omega}
\]

Relative eccentricity is determined by

\[
\varepsilon = k_1 S_0^{k_2}
\]

Minimum oil film thickness is given by

\[
h_0 = \left[1/2\right] DC(1 - \varepsilon)
\]

The minimum oil film thickness obtained by Mathcad software is 3.2449 \( \mu m \) and the KISSsoft result is 3.14 \( \mu m \). The percentage error between these two results is about 3.2328 \%.

Due to the use of a simple calculation method, it can be estimated that the relative error in the calculated minimum oil thickness was less, about \( \pm 5\% \), when the results are compared to values determined by detail calculation methods.

V. MATHCAD TO CREO INTEGRATION

Mathcad can be integrated with Creo Parametric. There is provision for integration in Creo Parametric in Analysis tab located in command bar of software. This section explains how to integrate Mathcad with Creo Parametric. Following process steps gives brief idea for integration of Mathcad to Creo Parametric by using crankshaft ball bearing, because it is difficult to simulate oil film thickness in journal bearing using Creo.

Create a ball bearing defined by outer and inner diameter with specified width in Creo Parametric.

Due to the use of a simple calculation method, it can be estimated that the relative error in the calculated minimum oil thickness was less, about \( \pm 5\% \), when the results are compared to values determined by detail calculation methods.
After loading Mathcad file, assign scale to 1 and the dimension which you want to change by integration. Here, we use bearing diameter to 48mm. Assign these parameters as input and output parameters. Then save the Mathcad worksheet.

Figure 8. Mathcad worksheet with 48 mm inner diameter.

Now go to Creo prime analysis interface. Tab shows the name of Mathcad file loaded in it. Click Auto-map and create Creo parameter to prime mapping. Right click and Select Creo parametric parameter. Parameter selection pop up and select scale parameter.

Figure 9. Parameter selection and mapping.

After this, right click and select prime input scale.

Figure 10. Mapping prime input.

Create prime to Creo parameter mapping. Right click and Select prime output parameter. Parameter selection pop up and select scale parameter as D.

Figure 11. Prime to Creo parameter mapping.

Now prime analysis tab shows mapped value and then click compute. Add feature name for analysis and close it.

Figure 12. Mapping results and computation.

Go to tools tab and add local parameters in relations and assign relation to parameter for verifying relations. Specify relations in feature parameter. Click on local parameter. Add parameter to relations and verify relations. Click ok.

Figure 13. Specifying relations and its verification.

The software gives yellow indication to complete regeneration step. Then right click on extrude and sketch command applied and edit definition for checking dimensions.

Figure 14. Regeneration of inner part to 48 mm diameter.

Then complete regeneration and check dimension of inner part of bearing. It will shows new dimension which applied in Mathcad worksheet. Here, diameter of bearing is modified using integration process from 50mm to 48mm.

Right click on ANALYSIS1 command applied and edit definition with clicking next for changing dimensions of Mathcad worksheet.

Click compute tab. The dimensions changes automatically. Regenerate the changing parameter step. Then check extrude and sketch dimension by editing definitions of both.

Figure 15. Bearing with modified inner diameter to 48 mm.
VI. CONCLUSION

The main result of this study was the mathematical formulation of realistic oil film thickness in real type hydrodynamic journal bearings at various operating points across the realistic operating range. This template can be used for other engine bearings, minimum oil film calculation.

Analytical calculations are highly complex and very difficult to perform using MS Excel. So Mathcad is a good solution for performing this type of complex calculations and formulations.

The minimum oil film thickness obtained by Mathcad template is 3.2449 µm and the KISSsoft result is 3.14 µm. The percentage error between these two results is about 3.2328 %.

After performing whole calculations, the template is used to integrate with actual CAD model of the components. In this paper, Integration process for ball bearing is shown in detail using Mathcad and Creo. Then actual CAD model will be automatically modified as dimension in Mathcad worksheet modifies after successful regeneration in Creo parametric. Mathcad template results are in good agreement with “KISSsoft” results.

APPENDIX

The relative eccentricity \( \varepsilon \) was determined approximately as a function of the Sommerfeld number \( S_o \) and the width-to-diameter ratio \( B / D \). The approximation of relative eccentricity was based on a graph which is presented in a standard for hydrodynamic journal bearings with a static load (DIN 31652 Teil 2, 1983) and in which the Sommerfeld number \( S_o \) is presented as a function of the width-to-diameter ratio \( B / D \) at different values of the relative eccentricity \( \varepsilon \). The estimated relative eccentricity in a typical case (with the width \( B = 32 \) mm, the diameter \( D = 85 \) mm, and the width-to-diameter ratio \( B / D = 0.376 \)) is presented in Figure 16.

![Figure 16. Estimated relative eccentricity \( \varepsilon \) as a function of the Sommerfeld number \( S_o \). The width-to-diameter ratio \( B / D = 0.376 \), and \( 1 < S_o < 200 \).](image)

ACKNOWLEDGMENT

The Author wish to acknowledge the support of Greaves Cotton Limited, during the whole period of study.

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